Chapter 3 : Square and Square Roots

ANSWER KEYS

EXERCISE 3.1

1. We know that the natural numbers ending with the digits 2, 3, 7 or 8 are not perfect squares.

(*i*) 54473 ends with digit 3.

(*ii*) 4058 ends with digit 8.

(*iii*) 24257 ends with digit 7.

(*iv*) 3332 ends with digit 2.

Therefore, all these numbers are not perfect squares.

2. If a number has 1 or 9 in the unit place, then its square ends with 1, if the number has 4 or 6 in the unit place, then its square ends with 6, and the number 3 or 7 in the unit place, its square ends with 9. Therefore,

(*i*) 1234 ends with 4, its square ends with 6.

(*ii*) 4329 ends with 9, its square ends with 1.

(*iii*) 8723 ends with 3, its square ends with 9.

3. The units digits of the square of a number having digits at units places as 1 or 9 is 1.

Therefore,

$(321)^2$ and $(549)^2$ will have 1 as their units digits.

Hence, (ii) and (iii) have 1 as their units digit.

4. (*i*) 2826 being an even number $(2826)^2$ will also be an even number.

(*ii*) 7779 being an odd number, $(7779)^2$ will also be on odd number.

(*iii*) 30018 being an even number, $(30018)^2$ will also be an even number.

(*iv*) 8204 being an even number, $(8204)^2$ will also be an even number.

5. Since, between n^2 and $(n + 1)^2$, there are $2n$ non-perfect square numbers. Therefore,

(*i*) $(25)^2$ and $(26)^2$ there are $2 \times 25 = 50$ natural number.

(*ii*) $(19)^2$ and $(20)^2$ there are $2 \times 19 = 38$ natural numbers.

6. (*i*) $(65)^2 = (6 \times 7)$ hundred + 25
$= 4225$

(*ii*) $(95)^2 = (9 \times 10)$ hundred + 25
$= 9025$

(*iii*) $(205)^2 = (20 \times 21)$ hundred + 25
$= 42025$

EXERCISE 3.2

1. We know that $2m$, $m^2 - 1$, $m^2 + 1$ is a Pythagorean triplet $2m = 8$

$\therefore\ m = 4$

$\therefore\ m^2 - 1 = (4)^2 - 1 = 16 - 1 = 15$

$m^2 + 1 = (4)^2 + 1 = 16 + 1 = 17$

Hence, 8, 15, 17 is a Pythagorean triplet.

2. Let $2m = 12$

$\therefore\ m = \frac{12}{2} = 6$

Now, $m^2 - 1 = (6)^2 - 1 = 36 - 1 = 35$

and $m^2 + 1 = (6)^2 + 1 = 36 + 1 = 37$

Hence 12, 35, 37, is a Pythagorean triplet.

3. Let $2m = 18$

$\Rightarrow\ m = 9$

$\therefore\ m^2 - 1 = (9)^2 - 1 = 81 - 1 = 80$

$m^2 + 1 = (9)^2 + 1 = 81 + 1 = 82$

Hence, 18,80,82 is Pythagorean triplet.

4. If a, b, c are three number where $c > a, b$ such that $a^2 + b^2 = c^2$, then (a, b, c) is called Pythagorean triplet.

Thus, $(26)^2 = 676$

$(10)^2 + (24)^2 = 100 + 576 = 676$

$\Rightarrow\ (26)^2 = (10)^2 + (24)^2$

Hence, 10, 24 and 26 is a Pythagorean triplet.

EXERCISE 3.3

1. (*i*) Since, ones digit of 6561 is 1, the possible ones digit of the square root may be 1 or 9.

(*ii*) Since, ones digit of 24336 is 6, the possible ones digit of the square root may be 4 or 6.

(*iii*) Since, ones digit of 76129 is 9, the possible ones digit of the square root may be 3 or 7.

(*iv*) Since, ones digit of 160801 is 1, the possible ones digit of the square root may be 1 or 9.

2. We know that if the units digit of a number is 2, 3, 7 or 8, then it does not have a square root in the set of natural numbers, hence it will not be a perfect square.

   Hence (*i*) 1267, (*ii*) 608 and (*iii*) 1990 is not a perfect squares.

   $\therefore \quad 180 = (2 \times 2) \times (3 \times 3) \times 5$

3. The number 5 is left unpaired.

2	180
2	90
3	45
3	15
3	5
	1

So, 180 is not a perfect square.
Now, If we multiplied with 5 it should become a perfect square. *ie*.

$180 \times 5 = (2 \times 2) \times (3 \times 3) \times (5 \times 5)$

$900 = (2 \times 2) \times (3 \times 3) \times (5 \times 5)$

Square root of 900 $= \sqrt{900}$

$= \sqrt{(2 \times 2) \times (3 \times 3) \times (5 \times 5)}$

$= 2 \times 3 \times 5 = 30$

Hence, square root of the new number is 30.

4. (*i*) We have

$144 - 1 = 143$
$143 - 3 = 140$
$140 - 5 = 135$
$135 - 7 = 128$
$128 - 9 = 119$
$119 - 11 = 108$
$108 - 13 = 95$
$95 - 15 = 80$
$80 - 17 = 63$
$63 - 19 = 44$
$44 - 21 = 23$
$23 - 23 = 0$

We have performed subtraction 12 times.
Hence, $\sqrt{144} = 12$

(*ii*) We have

$289 - 1 = 288$
$288 - 3 = 285$
$285 - 5 = 280$
$280 - 7 = 273$
$273 - 9 = 264$
$264 - 11 = 253$
$253 - 13 = 240$
$240 - 15 = 225$
$225 - 17 = 208$
$208 - 19 = 189$
$189 - 21 = 168$
$168 - 23 = 145$
$145 - 25 = 120$
$120 - 27 = 93$
$93 - 29 = 64$
$64 - 31 = 33$
$33 - 33 = 0$

We have performed subtraction 17 times.
Hence, $\sqrt{289} = 17$.

(*iii*) We have

$169 - 1 = 168$
$168 - 3 = 165$
$165 - 5 = 160$
$160 - 7 = 153$
$153 - 9 = 144$
$144 - 11 = 133$
$133 - 13 = 120$
$120 - 15 = 105$
$105 - 17 = 88$
$88 - 19 = 69$
$69 - 21 = 48$
$48 - 23 = 25$
$25 - 25 = 0$

We have performed subtraction 13 times.
Hence $\sqrt{169} = 13$.

5. (*i*) 676

$\therefore \quad 676 = 2 \times 2 \times 13 \times 13$

$\therefore \quad \sqrt{676} = \sqrt{\underline{2 \times 2} \times \underline{13 \times 13}}$

$= 2 \times 13$

$= 26$

2	676
2	338
13	169
13	13
	1

Hence, $\sqrt{676} = 26$

(*ii*) 1024

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$\therefore \quad 1024 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$\therefore \quad \sqrt{1024} = \sqrt{\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2}}$

$= 2 \times 2 \times 2 \times 2 \times 2$

$= 32$

Hence, $\sqrt{1024} = 32$

(*iii*) 27225

$27225 = 3 \times 3 \times 5 \times 5 \times 11 \times 11$

3	27225
3	9075
5	3025
5	605
11	121
11	11
	1

$\therefore \quad \sqrt{27225} = \sqrt{\underline{3 \times 3} \times \underline{5 \times 5} \times \underline{11 \times 11}}$

$= 3 \times 5 \times 11$

$= 165$

Hence, $\sqrt{27225} = 165$

(*iv*) 7744

$7744 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 11 \times 11$

2	7744
2	3872
2	1936
2	968
2	484
2	242
11	121
11	11
	1

$\sqrt{7444} = \sqrt{\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{11 \times 11}}$

$= 2 \times 2 \times 2 \times 11$

$= 88$

Hence, $\sqrt{7744} = 88$

(*v*) 9604

$9604 = 2 \times 2 \times 7 \times 7 \times 7 \times 7$

2	9604
2	4802
7	2401
7	343
7	49
7	7
	1

$\therefore \quad \sqrt{9604} = \sqrt{\underline{2 \times 2} \times \underline{7 \times 7} \times \underline{7 \times 7}}$

$= 2 \times 7 \times 7$

$= 98$

Hence, $\sqrt{9605} = 98$

(*vi*) 15625

$15625 = 5 \times 5 \times 5 \times 5 \times 5 \times 5$

5	15625
5	3125
5	625
5	125
5	25
5	5
	1

$\therefore \quad \sqrt{15625} = \sqrt{\underline{5 \times 5} \times \underline{5 \times 5} \times \underline{5 \times 5}}$

$= 5 \times 5 \times 5$

$= 125$

Hence, $\sqrt{15625} = 125$

(*vii*) 390625

$390625 = 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$

$\sqrt{390625} = \sqrt{\underline{5 \times 5} \times \underline{5 \times 5} \times \underline{5 \times 5} \times \underline{5 \times 5}}$

5	390625
5	78125
5	15625
5	3125
5	625
5	125
5	25
5	5
	1

$= 5 \times 5 \times 5 \times 5 = 625$

Hence, $\sqrt{390625} = 625$

(*viii*) 9801

$9801 = 3 \times 3 \times 3 \times 3 \times 11 \times 11$

3	9801
3	3267
3	1089
3	363
11	121
11	11
	1

$\sqrt{9801} = \sqrt{\underline{3 \times 3} \times \underline{3 \times 3} \times \underline{11 \times 11}}$

$= 3 \times 3 \times 11$

$= 99$

Hence, $\sqrt{9801} = 99$

(*ix*) 99856

$99856 = 2 \times 2 \times 2 \times 2 \times 79 \times 79$

2	99856
2	49928
2	24964
2	12482
79	6241
79	79
	1

$\sqrt{99856} = \sqrt{\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{79 \times 79}}$

$= 2 \times 2 \times 79$

$= 316$

Hence, $\sqrt{99856} = 316$

(*x*) 11025

$11025 = 3 \times 3 \times 5 \times 5 \times 7 \times 7$

3	11025
3	3675
5	1225
5	245
7	49
7	7
	1

$\sqrt{11025} = \sqrt{\underline{3 \times 3} \times \underline{5 \times 5} \times \underline{7 \times 7}}$

$= 3 \times 5 \times 7$

$= 105$

Hence, $\sqrt{11025} = 105$

6. $2352 = (2 \times 2) \times (2 \times 2) \times (7 \times 7) \times 3$

The number 3 is left unpaired. So, it is not a perfect square. If we multiplied by 3 it should become a perfect square. So,

$2352 \times 3 = (2 \times 2) \times (2 \times 2) \times (7 \times 7) \times (3 \times 3)$

$7056 = (2 \times 2) \times (2 \times 2) \times (7 \times 7) \times (3 \times 3)$

2	2352
2	1176
2	588
2	294
7	147
7	21
3	3
	1

$\sqrt{7056} = \sqrt{\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{7 \times 7} \times \underline{3 \times 3}}$

$= 2 \times 2 \times 7 \times 3 = 84$

Hence, $\sqrt{7056} = 84$

7. (*i*) 252

$252 = (2 \times 2) \times (3 \times 3) \times 7$

The number 7 is left unpaired.

If multiplied by 7 it should be a perfect square number. So,

$252 \times 7 = (2 \times 2) \times (3 \times 3) \times (7 \times 7)$

2	252
2	126
3	63
3	21
7	7
	1

$\sqrt{1764} = \sqrt{\underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}}$

$= 2 \times 3 \times 7 = 42$

$\sqrt{1764} = 42$

Hence, if multiplied by 7, then the given number makes a perfect square and its square root is 42.

(*ii*) 1458

$1458 = 2 \times (3 \times 3) \times (3 \times 3) \times (3 \times 3)$

The number 2 is left unpaired.

If multiplied by 2, it should be a perfect square number. So,

$(1458 \times 2) = (2 \times 2) \times (3 \times 3) \times (3 \times 3) \times (3 \times 3)$

$2916 = (2 \times 2) \times (3 \times 3) \times (3 \times 3) \times (3 \times 3)$

2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

$\sqrt{2916}$

$= \sqrt{\underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3} \times \underline{3 \times 3}}$

$= 2 \times 3 \times 3 \times 3$

$\sqrt{2916} = 54$

Hence, if multiplied by 2, then the given number make a perfect square and its square root is 54.

(*iii*) 768

$768 = (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times 3$

The number 3 is left unpaired.

If we multiplied by 3, it should be a perfect square number. So,

$768 \times 3 = (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (3 \times 3)$

$2304 = (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (3 \times 3)$

2	768
2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

$\sqrt{2304} = \sqrt{\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3}}$

$\sqrt{2304} = 2 \times 2 \times 2 \times 2 \times 3$

$\sqrt{2304} = 48$

Hence, if multiplied by 3, then the given number make a perfect square number and its square root is 48.

8. We know that, if the units digit of a number is 2, 3, 7 or 8, then it does not a perfect square number.

Therefore,

(*i*) 1053, (*ii*) 628 and (*iii*) 657 are not a perfect square number.

While 3481 is a perfect square number.

EXERCISE 3.4

1. The least number divisible by each of the numbers 6, 9 and 15 is their L.C.M.

L.C.M of 6, 9 and 15 is

3	6,9,15
	2, 3, 5

$3 \times 2 \times 3 \times 5 = 90$

Now, $90 = 2 \times (3 \times 3) \times 5$

The numbers 2 and 5 are not in pair for the number to be a perfect square each factor of the must have a pair. So,

To make pairs of 2 and 5, the number 90 has to be multiplied by 5×2 *ie*. 10.

Hence, $90 \times 10 = 900$ is the required square number.

2. $5392 = (2 \times 2) \times (2 \times 2) \times 337$

The number 337 is left unpaired. So the given number must be divided by 337 to get a square number.

2	5392
2	2696
2	1348
2	674
	337

$\therefore \quad \frac{5392}{337} = 16$

Now, $\sqrt{16} = \sqrt{\underline{2 \times 2} \times \underline{2 \times 2}} = 4$

Hence, square root of new number is 4.

3. The least number which is divisible by each of the numbers 4, 9 and 10 is their L.C.M.

L.C.M. of 4, 9 and 10

$= 2 \times 2 \times 9 \times 5$

$= 180$

2	4,9,10
	2,9, 5

Now, $180 = (2 \times 2) \times (3 \times 3) \times 5$

The number 5 is left unpaired. For the number to be a perfect square, each factor of the number must have a pair, to make pair of the number, 180 has to be multiplied by 5.

Hence, the required number is $180 \times 5 = 900$.

4. The prime factors of 2925

$= (3 \times 3) \times (5 \times 5) \times 13$

The number 13 is left unpaired.

So, the given number must be divided by 13 to get a square number.

3	2925
3	975
5	325
5	65
13	13
	1

$\therefore \frac{2925}{13} = 225$ is a square number.

Hence, 13 is the required smallest number.

5. Let the one number be x, then the other number will be $13x$.

Then, $x \times 13x = 2197$

$\Rightarrow 13x^2 = 2197$

$\Rightarrow x^2 = \frac{2197}{13} = 169$

$\Rightarrow x = \sqrt{169}$

$= \sqrt{13 \times 13}$

$x = 13$

13	169
13	13
	1

$\therefore$ One number is 13 and the other number is 169. Hence, required numbers are 13 and 169.

6. Let the number be x.

$\therefore$ Other number will be $3x$.

Then, $x \times 3x = 5292$

$\Rightarrow 3x^2 = 5292$

$\Rightarrow x^2 = \frac{5292}{3} = 1764$

$\Rightarrow x = \sqrt{1764}$

$x = \sqrt{\underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}}$

$x = 2 \times 3 \times 7$

$x = 42$

2	1764
2	882
3	441
3	147
7	49
7	7
	1

$\therefore$ One number is 42 and the other number is 126. Hence, required numbers are 42 and 126.

7. Length of a rectangular field = 81 m

Breadth of a rectangular field = 49 m

$\therefore$ Area of rectangular field = length × breadth

= 81m × 49m

= $(81 \times 49)\text{m}^2$

= 3969 m^2

Since, area of square = area of rectangular field

$\Rightarrow (\text{Side})^2 = 3969 \text{ m}^2$

$\Rightarrow \text{side} = \sqrt{3969}\text{m}$

$= \sqrt{\underline{3 \times 3} \times \underline{3 \times 3} \times \underline{7 \times 7}}$ m

= $(3 \times 3 \times 7)$m

= 63 m

3	3969
3	1323
3	441
3	147
7	49
7	7
	1

Hence, side of the square is 63 m.

8. Let number of students in a school be x, Then,

$x \times x = 11025$

$\Rightarrow x^2 = 11025$

$\Rightarrow x = \sqrt{11025}$

$\Rightarrow x = \sqrt{\underline{3 \times 3} \times \underline{5 \times 5} \times \underline{7 \times 7}}$

$x = 3 \times 5 \times 7$

$x = 105$

3	11025
3	3675
5	1225
5	245
7	49
7	7
	1

Hence, there were 105 students in the school.

9. $27783 = (3 \times 3) \times (3 \times 3) \times (7 \times 7) \times 7$

The number 7 is left unpaired.

So, 7 is multiplied to the number to make a perfect square number.

3	27783
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

10. Let the number be x. Then, the other number is $\frac{2205}{x}$.

Therefor, $\frac{x}{\frac{2205}{x}} = \frac{9}{5}$

$\Rightarrow \frac{x^2}{2205} = \frac{9}{5}$

$\Rightarrow x^2 = \frac{9}{5} \times 2205$

$= 9 \times 441$

$x^2 = 3969$

3	3969
3	1323
3	441
3	147
7	49
7	7
	1

$\Rightarrow \quad x = \sqrt{3969}$

$x = \sqrt{\underline{3 \times 3} \times \underline{3 \times 3} \times \underline{7 \times 7}}$

$= 3 \times 3 \times 7$

$x = 63$

Hence, the required numbers are 63 and 35.

11. Let the number be x. Then, the other number is $\frac{4046}{x}$. Therefore,

$$\frac{x}{\frac{4046}{x}} = \frac{7}{2}$$

$$\Rightarrow \quad \frac{x^2}{4046} = \frac{7}{2}$$

$$x^2 = \frac{7}{2} \times 4046$$

$$= 7 \times 2023$$

$$x^2 = 14161$$

7	14161
7	2023
17	289
17	17
	1

$$\Rightarrow \quad x^2 = 7 \times 7 \times 17 \times 17$$

$$x = \sqrt{\underline{7 \times 7} \times \underline{17 \times 17}}$$

$$x = 7 \times 17$$

$$x = 119$$

Hence, the required numbers are 119 and 34.

3	2925
3	975
5	325
5	65
13	13
	1

12. (*i*) 2925

$2925 = (3 \times 3) \times (5 \times 5) \times 13$

The number 13 is left unpaired.

So, the given is divided by 13 to get a square number.

So, $\frac{2925}{13} = 225$

3	225
3	75
5	25
5	5
	1

$\sqrt{225} = \sqrt{\underline{3 \times 3} \times \underline{5 \times 5}}$

$\sqrt{225} = 3 \times 5 = 15$

(*ii*) 2800

$2800 = (2 \times 2) \times (2 \times 2) \times (5 \times 5) \times 7$

The number 7 is left unpaired.

So, the given number is divided by 7 to get a square number. So,

$\therefore \frac{2800}{7} = 400$ is a square number.

2	2800
2	1400
2	700
2	350
5	175
5	35
7	7
	1

2	400
2	200
2	100
2	50
5	25
5	5
	1

$\sqrt{400} = \sqrt{\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{5 \times 5}}$

$\sqrt{400} = 2 \times 2 \times 5$

$= 20$

(*iii*) 2645

$2645 = 5 \times (23 \times 23)$

The number 5 is left unpaired. So, the given number is divided by 5 to get a square number.

5	2645
23	529
23	23
	1

$\therefore \frac{2645}{5} = 529$

23	529
23	23
	1

$\sqrt{529} = \sqrt{23 \times 23}$

$\sqrt{529} = 23$

EXERCISE 3.5

1.

	9 9
9	$\overline{98}\,\overline{01}$
	8 1
189	1 7 0 1
	1 7 0 1
	0

$\therefore \sqrt{9801} = 99$

2.

	2 3 4
2	$\overline{05}\,\overline{47}\,\overline{56}$
	4
43	1 4 7
	1 2 9
464	1 8 5 6
	1 8 5 6
	0

$\therefore \sqrt{54756} = 234$

3.

	9 3
9	$\overline{86}\,\overline{49}$
	8 1
183	5 4 9
	5 4 9
	0

$\therefore \quad \sqrt{8649} = 93$

4.

	6 7
6	$\overline{44}\,\overline{89}$
	3 6
127	8 8 9
	8 8 9
	0

$\therefore \quad \sqrt{4489} = 67$

5.

	5 7
5	$\overline{32}\,\overline{49}$
	2 5
107	7 4 9
	7 4 9
	0

$\therefore \quad \sqrt{3249} = 57$

6.

	1 0 1
1	$\overline{01}\,\overline{02}\,\overline{01}$
	1
201	0 2 0 1
	2 0 1
	0

$\therefore \quad \sqrt{10201} = 101$

7.

	1 1 0
1	$\overline{01}\,\overline{21}\,\overline{00}$
	1
21	2 1
	2 1
	0 0
	0 0
	0

$\therefore \quad \sqrt{12100} = 110$

8.

	1 6 5
1	$\overline{02}\,\overline{72}\,\overline{25}$
	1
26	1 7 2
	1 5 6
325	1 6 2 5
	1 6 2 5
	0

$\therefore \quad \sqrt{27225} = 165$

9.

	1 4 3
1	$\overline{02}\,\overline{04}\,\overline{49}$
	1
24	1 0 4
	9 6
283	8 4 9
	8 4 9
	0

$\therefore \quad \sqrt{20449} = 143$

10.

	1 7 1
1	$\overline{02}\,\overline{92}\,\overline{41}$
	1
27	1 9 2
	1 8 9
341	3 4 1
	3 4 1
	0

$\therefore \quad \sqrt{29241} = 171$

11.

	1 6 3
1	$\overline{02}\,\overline{65}\,\overline{69}$
	1
26	1 6 5
	1 5 6
323	9 6 9
	9 6 9
	0

$\therefore \quad \sqrt{26569} = 163$

12.

	2 3 9
2	$\overline{05}\,\overline{71}\,\overline{21}$
	4
43	171
	129
469	4221
	4221
	0

$\therefore \quad \sqrt{57121} = 239$

EXERCISE 3.6

1. (*i*) $\sqrt{36864} = \sqrt{\overline{03}\,\overline{68}\,\overline{64}}$, square root will have 3 digits.

(*ii*) $\sqrt{28900} = \sqrt{\overline{02}\,\overline{89}\,\overline{00}}$, square root will have 3 digits.

(*iii*) $\sqrt{106276} = \sqrt{\overline{10}\,\overline{62}\,\overline{76}}$, square root will have 3 digits.

(*iv*) $\sqrt{4507129} = \sqrt{\overline{04}\,\overline{50}\,\overline{71}\,\overline{29}}$, square root will have 4 digits.

(*v*) $\sqrt{32400} = \sqrt{\overline{03}\,\overline{24}\,\overline{00}}$, square root will have 3 digits.

(*vi*) $\sqrt{5625} = \sqrt{\overline{56}\,\overline{25}}$, square root will have 2 digits.

(*vii*) $24336 = \sqrt{\overline{02}\,\overline{43}\,\overline{36}}$, square root will have 3 digits.

2. The greatest five digit number is 99999. Let us find square root of 99999.

From the square root of 99999, we can notice that $(316)^2$ is less then 99999 by 143.

If we subtracted the remainder 143 from the number, we get a perfect square number.

	3 1 6
3	$\overline{09}\,\overline{99}\,\overline{99}$
	9
61	99
	61
626	3899
	3756
	143

$\therefore$ 99999 – 143 = 99856 is the required number.

Hence, the greatest five digit number is 99856

3. Least number of four-digits is 1000.

	3 1
6	$\overline{10}\,\overline{00}$
	9
61	100
	61
	39

also, $(32)^2 = 1024$

Now, $(32)^2 - 1000 = 1024 - 1000 = 24$

We notice that $(31)^2 < 1000$, Thus, if we added 24 to 1000, it becomes a perfect square.

Hence, the smallest four digit square number is 1024.

4. (*i*)

	7 1
7	$\overline{50}\,\overline{45}$
	49
141	145
	141
	4

$(71)^2 < 5045$ by 4. So, in order to get a perfect square number we subtract 4 from 5045.

(*ii*)

	1 3 5
1	$\overline{01}\,\overline{82}\,\overline{65}$
	1
23	82
	69
265	1365
	1325
	40

$(135)^2$ is less than 18265 by 40. So, in order to get a perfect square number, we subtract 40 from 18265.

(*iii*)

	4 4 1
4	$\overline{19}\,\overline{44}\,\overline{91}$
	16
84	344
	336
881	891
	881
	10

$(441)^2$ is less than 194491 by 10. So, in order to get a perfect square number, we subtracted 10 from 194491.

(*iv*)

$$\begin{array}{r|l} & \;\;1\;\;\;6\;\;\;2 \\ 1 & \overline{02}\,\overline{65}\,\overline{35} \\ & \;\;1 \\ \hline 26 & 165 \\ & 156 \\ \hline 322 & \quad 935 \\ & \quad 644 \\ \hline & \quad 291 \end{array}$$

$(162)^2 < 26535$ by 291. So, in order to get a perfect square number, we subtracted 291 from 26535.

5. (i) The given number is 3720.

$$\begin{array}{r|l} & \;6\;\;\;1 \\ 6 & \overline{37}\,\overline{20} \\ & 36 \\ \hline 12 & \;\;120 \\ & \;\;\boxed{121} \end{array}$$

$(60)^2 = 3600$, $(61)^2 = 3721$

$(61)^2 > 3720 > (60)^2$

Thus, $(61)^2 - 3720 = 3721 - 3720 = 1$

Hence, the number to be added is 1.

(ii) The given number is 115580.

$$\begin{array}{r|l} & \;3\;\;\;3\;\;\;9 \\ 3 & \overline{11}\,\overline{55}\,\overline{80} \\ & \;\;9 \\ \hline 63 & 255 \\ & 189 \\ \hline 669 & \;\;6680 \\ & \;\;6021 \\ \hline & \quad 659 \end{array}$$

Also $(339)^2 = 114921$

$(340)^2 = 115600$

$(340)^2 > 115580 > (339)^2$

Thus, $(340)^2 - 115580 = 115600 - 115580 = 20$

Hence, the number to be added is 20.

(*iii*) The given number is 4931.

$$\begin{array}{r|l} & \;7\;\;\;0 \\ 7 & \overline{49}\,\overline{31} \\ & 49 \\ \hline 14 & \;\;\;31 \end{array}$$

Since,

$(70)^2 = 4900$ and $(71)^2 = 5041$.

$\therefore$ $(71)^2 > 4931 > (70)^2$

Thus, $(71)^2 - 4931 = 5041 - 4931 = 110$

Hence, the given number to be added is 110.

EXERCISE 3.7

1. (*i*) $\sqrt{\dfrac{16}{25}} = \dfrac{\sqrt{16}}{25}$

$$\begin{array}{r|l} & \;\;4 \\ 4 & \overline{16} \\ & 16 \\ \hline & \;\;0 \end{array} \qquad \begin{array}{r|l} & \;\;5 \\ 5 & \overline{25} \\ & 25 \\ \hline & \;\;0 \end{array}$$

$= \dfrac{4}{5}$

Hence, $\sqrt{\dfrac{16}{25}} = \dfrac{4}{5}$

(*ii*) $\sqrt{6\dfrac{1}{4}} = \sqrt{\dfrac{25}{4}}$

$$\begin{array}{r|l} & \;\;5 \\ 5 & \overline{25} \\ & 25 \\ \hline & \;\;0 \end{array} \qquad \begin{array}{r|l} & \;\;2 \\ 2 & \overline{04} \\ & \;\;4 \\ \hline & \;\;0 \end{array}$$

$= \dfrac{5}{4}$

Hence, $\sqrt{6\dfrac{1}{4}} = \dfrac{5}{4}$

(*iii*) $\sqrt{27\dfrac{1}{25}} = \sqrt{\dfrac{676}{25}}$

$$\begin{array}{r|l} & \;2\;\;\;6 \\ 2 & \overline{06}\,\overline{76} \\ & \;\;4 \\ \hline 46 & \;\;276 \\ & \;\;276 \\ \hline & \;\;\;\;0 \end{array} \qquad \begin{array}{r|l} & \;\;5 \\ 5 & \overline{25} \\ & 25 \\ \hline & \;\;0 \end{array}$$

$= \dfrac{26}{5}$

Hence, $\sqrt{27\dfrac{1}{25}} = \dfrac{26}{5}$

(*iv*) $\sqrt{7\frac{9}{16}} = \sqrt{\frac{121}{16}}$

	1 1
1	$\overline{1}\ \overline{21}$
	1
21	2 1
	2 1
	0

	4
4	$\overline{16}$
	1 6
	0

$= \frac{11}{4}$

Hence, $\sqrt{7\frac{9}{10}} = \frac{11}{4}$

2. (*i*) The given fraction is $\frac{80}{405}$. On simplifying,

$\frac{80}{405} = \frac{16}{81}$

$\sqrt{\frac{80}{405}} = \sqrt{\frac{16}{81}} = \frac{\sqrt{16}}{\sqrt{81}}$

	4
4	$\overline{16}$
	1 6
	0

	9
9	$\overline{81}$
	8 1
	0

$\sqrt{\frac{80}{405}} = \sqrt{\frac{16}{81}} = \frac{4}{9}$

(*ii*) $\sqrt{\frac{1225}{12321}} = \frac{\sqrt{1225}}{\sqrt{12321}}$

	3 5
3	$\overline{12}\ \overline{25}$
	9
65	3 2 5
	3 2 5
	0

	1 1 1
1	$\overline{01}\ \overline{23}\ \overline{21}$
	1
21	2 3
	2 1
221	2 2 1
	2 2 1
	0

Hence, $\sqrt{\frac{1225}{12321}} = \frac{35}{111}$

(*iii*) $\sqrt{1\frac{155}{169}} = \sqrt{\frac{324}{169}}$

	1 8
1	$\overline{03}\ \overline{24}$
	1
28	2 2 4
	2 2 4
	0

	1 3
1	$\overline{01}\ \overline{69}$
	1
23	6 9
	6 9
	0

Hence, $\sqrt{1\frac{155}{169}} = \frac{18}{13}$

(*iv*) $\sqrt{6\frac{145}{256}} = \sqrt{\frac{1681}{256}}$

	4 1
4	$\overline{16}\ \overline{81}$
	1 6
81	8 1
	8 1
	0

	1 6
1	$\overline{02}\ \overline{56}$
	1
26	1 5 6
	1 5 6
	0

Hence, $\sqrt{6\frac{145}{256}} = \frac{41}{16}$

(*v*) $\sqrt{84\frac{37}{121}} = \sqrt{\frac{10201}{121}}$

	1 0 1
1	$\overline{01}\ \overline{02}\ \overline{01}$
	1
201	0 2 0 1
1	2 0 1
	0

	1 1
1	$\overline{01}\ \overline{21}$
	1
21	2 1
	2 1
	0

Hence, $\sqrt{84\frac{37}{121}} = \frac{101}{11}$

(*vi*) $\sqrt{\frac{1183}{2023}} = \sqrt{\frac{169}{289}}$ (By Simplification)

	1 3
1	$\overline{01}\ \overline{69}$
	1
23	6 9
	6 9
	0

	1 7
1	$\overline{02}\ \overline{89}$
	1
27	1 8 9
	1 8 9
	0

Hence, $\sqrt{\frac{1183}{2023}} = \frac{13}{17}$

3. Area of a square field = $(\text{side})^2$

$\Rightarrow (\text{side})^2 = 23\frac{394}{729}$ sq.m

$\Rightarrow \quad \text{side} = \sqrt{23\frac{394}{729}} = \sqrt{\frac{17161}{729}}$

	1 3 1
1	$\overline{01}\,\overline{71}\,\overline{61}$
	1
23	71
	69
261	261
	261
	0

	2 7
2	$\overline{07}\,\overline{29}$
	4
47	329
	329
	0

Hence, side of a square field $= \frac{131}{27}\text{m} = 4\frac{23}{27}\text{m}$

4. $\because$ side of the square field $= \sqrt{\text{Area}}$

$= \sqrt{35\frac{1}{44}} = \sqrt{\frac{5041}{144}}$

	7 1
7	$\overline{50}\,\overline{41}$
	49
141	141
	141
	0

	1 2
1	$\overline{01}\,\overline{44}$
	1
22	44
	44
	0

Hence, side of the square field $= \frac{71}{12}\text{m} = 5\frac{11}{12}\text{m}$

5. (*i*) $\sqrt{80\frac{244}{729}} = \sqrt{\frac{58564}{729}} = \frac{\sqrt{58564}}{\sqrt{729}}$

	2 4 2
2	$\overline{05}\,\overline{85}\,\overline{64}$
	4
44	185
	176
482	964
	964
	0

	2 7
2	$\overline{07}\,\overline{29}$
	4
47	329
	329
	0

Hence, $\sqrt{80\frac{244}{729}} = \frac{242}{27} = 8\frac{26}{27}$

(*ii*) $\sqrt{75\frac{46}{49}} = \sqrt{\frac{3721}{49}} = \frac{\sqrt{3721}}{\sqrt{49}}$

	6 1
6	$\overline{37}\,\overline{21}$
	36
121	121
	121
	0

	7
7	$\overline{49}$
	49
	0

Hence, $\sqrt{75\frac{46}{49}} = \frac{61}{7} = 8\frac{5}{7}$

(*iii*) $\sqrt{\frac{841}{1024}} = \frac{\sqrt{841}}{\sqrt{1024}}$

	2 9
2	$\overline{08}\,\overline{41}$
	4
49	441
	441
	0

	3 2
3	$\overline{10}\,\overline{24}$
	9
62	124
	124
	0

Hence, $\sqrt{\frac{841}{1024}} = \frac{29}{32}$

6. (*i*) $\frac{\sqrt{441}+\sqrt{169}}{\sqrt{441}-\sqrt{169}}$

	2 1
2	$\overline{04}\,\overline{41}$
	4
41	41
	41
	0

	1 3
1	$\overline{01}\,\overline{69}$
	1
23	69
	69
	0

Thus, $\frac{\sqrt{441}+\sqrt{169}}{\sqrt{441}-\sqrt{169}} = \frac{21+13}{21-13} = \frac{34}{8} = \frac{17}{4}$

(*ii*) $\frac{\sqrt{576}+\sqrt{196}}{\sqrt{576}-\sqrt{196}}$

	2 4
2	$\overline{05}\,\overline{76}$
	4
44	176
4	176
	0

	1 4
1	$\overline{01}\,\overline{96}$
	1
24	96
	96
	0

$\sqrt{576} = 24 \qquad \sqrt{196} = 14$

Thus,

$$\frac{\sqrt{576}+\sqrt{196}}{\sqrt{576}-\sqrt{196}} = \frac{24+14}{24-14} = \frac{38}{10} = \frac{19}{5}$$

7. (*i*) $\sqrt{\frac{720}{1125}} = \sqrt{\frac{144}{225}} = \sqrt{\frac{144}{225}}$ (On simplification)

	1 2
1	$\overline{01}\,\overline{44}$
	1
22	44
	44
	0

	1 5
1	$\overline{02}\,\overline{25}$
	1
25	125
	125
	0

Thus, $\sqrt{\dfrac{720}{1125}} = \dfrac{12}{15}$

(ii) $\sqrt{\dfrac{1521}{1764}} = \sqrt{\dfrac{169}{196}}$

	13
1	169
	1
23	69
	69
	0

	14
1	01 96
	1
24	96
4	96
	0

Thus, $\sqrt{\dfrac{1521}{1764}} = \dfrac{13}{14}$

EXERCISE 3.8

1. (*i*)

	2. 2 5
2	05.06 25
	4
42	106
	84
445	2225
	2225
	0

Hence, $\sqrt{5.0625} = 2.25$

(*ii*)

	5.6
5	31.36
	25
106	636
	636
	0

Hence, $\sqrt{31.36} = 5.6$

(*iii*)

	7.2
7	51.84
	49
142	284
	284
	0

Hence, $\sqrt{51.84} = 7.2$

(*iv*)

	9.8
9	98.01
	81
189	1701
	1701
	0

Hence, $\sqrt{98.01} = 9.8$

(*v*)

	0 . 3 9
0	00. 15 21
	00
3	15
	9
69	621
	621
	0

Hence, $\sqrt{0.1521} = 0.39$

(*vi*)

	2 2.3
2	04 97.29
	4
42	97
	84
443	1329
	1329
	0

Hence, $\sqrt{497.29} = 22.3$

2. (*i*)

	1.73205
1	03.00 00 00 00 00
	1
27	2 00
	1 89
343	1100
	1029
3462	7100
	6924
346405	1760000
	1732025
	27975

Hence, $\sqrt{3} = 1.73205$

$\sqrt{3} = 1.732$

(*ii*)

	2.6457
2	07.00 00 00 00
	4
46	3 00
	2 76
524	24 00
	20 96
5285	3 04 00
	2 64 25
5290	39 75 00
	37 03 49
	2 71 51

Hence, $\sqrt{7} = 2.6457$

$\sqrt{7} = 2.646$

(*iii*)

	3.3166
3	11.00 00 00 00
	9
63	2 00
	1 89
661	11 00
	6 61
6626	4 39 00
	3 97 56
66326	41 44 00
	39 79 56
	1 64 44

Hence, $\sqrt{11} = 3.3166$

$\sqrt{11} = 3.317$

3. (*i*)

	0.9486
0	0.90 00 00 00
	0
9	90
	81
184	9 00
	7 36
1888	1 64 00
	1 51 04
18966	12 96 00
	11 37 96
	1 58 04

Hence, $\sqrt{0.9} = 0.9486$

or $\sqrt{0.9} = 0.949$

(*ii*) $\sqrt{\frac{7}{8}} = \sqrt{0.875}$

	0.9354
0	0.87 50 00 00
	0
9	87
	81
183	6 50
	5 49
1865	1 01 00
	93 25
18704	7 75 00
	7 48 16
	26 84

$\therefore \sqrt{\frac{7}{8}} = 0.9354$

Hence, $\sqrt{\frac{7}{8}} = 0.935$

(*iii*) $\sqrt{4\frac{2}{3}} = \sqrt{\frac{14}{3}} = \sqrt{4.66666666}$

	2.1602
2	04.66 66 66 66
	4
41	66
	41
426	25 66
	25 56
43202	10 66 66
	8 64 04
	2 02 62

$\therefore \sqrt{4\frac{2}{3}} = \sqrt{4.666666} = 2.1602$

Hence, $\sqrt{4\frac{2}{3}} = 2.160$

(*iv*) $\sqrt{\frac{3}{7}} = \frac{\sqrt{3}}{\sqrt{7}}$

We have, $\sqrt{3}$ = 1.732 and $\sqrt{7}$ = 2.646

$\therefore \sqrt{\frac{3}{7}} = \frac{1.732}{2.646} = 0.6545$

Hence, $\sqrt{\frac{3}{7}} = 0.655$

4. (*i*) $\sqrt{0.021609}$

	0.147
0	0.02 16 09 00
	0
1	02
	1
24	116
	96
287	2009
	2009
	0

Hence, $\sqrt{0.021609} = 0.147$

(*ii*) $\sqrt{0.00155236}$

	0.0394
0	00.00 15 52 36
	0
0	00
	00
3	15
	9
69	652
	621
784	3136
	3136
	0

Hence, $\sqrt{0.00155236} = 0.0394$

5. Let the fraction be x. Then

$x \times x = 0.1764$

$\Rightarrow \quad x^2 = 0.1764$

$x = \sqrt{0.1764}$

$x = 0.42$

$= \frac{42}{100} = \frac{21}{50}$

	0.42
0	0.17 64
	0
4	17
	16
82	164
	164
	0

Hence, the required fraction is $\frac{21}{50}$.

6. Let the fraction be x. Then,

$x \times x = 0.7225$

$\Rightarrow \quad x^2 = 0.7225$

$x = \sqrt{0.7225}$

$x = 0.85$

$= \frac{85}{100} = \frac{17}{20}$

	0.85
0	0.72 25
	0
8	72
	64
165	825
	825
	0

Hence, required fraction is $\frac{17}{20}$.

7. Area of a square field = 1892.25 sq.m

$\therefore$ Area of a square field = $(\text{Side})^2$

$\Rightarrow$ $(\text{side})^2 = 1892.25$

side = $\sqrt{1892.25}$

	43.5
4	18 92.25
	16
83	2 92
	2 49
865	43 25
	43 25
	0

$\therefore$ side = 43.25 m

Hence, side of the square field = 43.25 m

8. Area of a square field = 37056.25 sq.m

$\therefore$ Area of square field = $(\text{Side})^2$

$\Rightarrow$ $(\text{side})^2 = 37056.25$

$\Rightarrow$ side = $\sqrt{37056.25}$

	192.5
1	03 70 56.25
	1
29	2 70
	2 61
382	956
	764
3845	192 25
	192 25
	0

$\therefore$ side = 192.50 m

Hence, side of the square field = 192.50 m

EXERCISE 3.9

1. $\sqrt{29}$

Look at the table, the entry in the column of $\sqrt{29}$ is 5.385.

$\therefore \quad \sqrt{29} = 5.385$.

2. $\sqrt{47}$

Look at the table, the entry in the column of $\sqrt{47}$ is 6.856

$\therefore \quad \sqrt{47} = 6.856$.

3. $\sqrt{78}$

Look at the table, the entry in the column of $\sqrt{78}$ is 8.832.

$\therefore \quad \sqrt{78} = 8.832 \cdot$

4. $\sqrt{84}$

Look at the table, the entry in the column of $\sqrt{84}$ is 9.165.

$\therefore \quad \sqrt{84} = 9.165 \cdot$

5. $\sqrt{1183} = \sqrt{169 \times 7} = 13\sqrt{7} \quad (\therefore \sqrt{169} = 13)$

$= 13 \times 2.646$

(from table $\sqrt{7} = 2.646$)

$= 34.398$

Hence, $\sqrt{1183} = 34.398$

6. $\sqrt{405} = \sqrt{81 \times 5} = 9\sqrt{5} \quad (\therefore \sqrt{81} = 9)$

$= 9 \times 2.236$

(from table $\sqrt{5} = 2.236$)

$= 20.124$

Hence, $\sqrt{405} = 20.124$

7. $\sqrt{801} = \sqrt{9 \times 89} = 3\sqrt{89}$

$= 3 \times 9.434$

(from table $\sqrt{89} = 9.434$)

$= 28.302$

Hence, $\sqrt{801} = 28.302$

8. $\sqrt{250} = \sqrt{25 \times 10} = 5\sqrt{10}$

$= 5 \times 3.162$

(from table $\sqrt{10} = 3.162$)

$= 15.81$

Hence, $\sqrt{250} = 18.81$

9. $\sqrt{378} = \sqrt{9 \times 42} = 3\sqrt{42}$

$= 3 \times 6.481$

(from table $\sqrt{42} = 6.481$)

$= 19.443$

Hence, $\sqrt{378} = 19.443$

10. $\sqrt{15.21} = \sqrt{\frac{1521}{100}} = \frac{\sqrt{9 \times 169}}{\sqrt{100}}$

$= \frac{\sqrt{9} \times \sqrt{169}}{10}$

$= \frac{3.00 \times 13.00}{10}$

$= \frac{39}{10} = 3.9$

Hence, $\sqrt{15.21} = 3.9$

11. $\sqrt{21.92} = \sqrt{\frac{2192}{100}}$

$= \sqrt{\frac{16 \times 137}{100}}$

$= \frac{4}{10}\sqrt{137}$

$= \frac{4}{10} \times 11.705$

(from table $\sqrt{137} = 11.705$)

$= \frac{4 \times 11.705}{10} = 4.682$

$= 4.68$

Hence, $\sqrt{21.92} = 4.68$

12. $\sqrt{13.14} = \sqrt{\frac{1314}{100}} = \sqrt{\frac{9 \times 146}{100}}$

$= \frac{3}{10}\sqrt{146}$

$= \frac{3}{10}\left(\sqrt{2} \times \sqrt{73}\right)$

$= \frac{3}{10}(1.414 \times 8.544)$

(from table $\sqrt{2} = 1.414$ & $\sqrt{73} = 8.544$)

$= \frac{3}{10} \times 12.081$

$= \frac{36.244}{10} = 3.624$

Hence, $\sqrt{13.14} = 3.62$

13. $\sqrt{83.17} = \sqrt{\frac{8317}{100}}$

For $\sqrt{83.47}$, we find approximate difference between $\sqrt{84}$ and $\sqrt{83}$.

$\sqrt{84} = 9.165$ and $\sqrt{85} = 9.110$ (from table)

$\sqrt{84} - \sqrt{83} = 9.165 - 9.110 = 0.055$

For the difference of 1 (= 84 – 83), the difference between $\sqrt{84}$ and $\sqrt{83} = 0.055$

For the difference 0.47 = 0.055 × 0.47 = 0.0259

$\therefore \quad \sqrt{83.47} = 9.110 + 0.0259$

$= 9.1359$

Hence, $\sqrt{83.17} = 9.14$

14. $\sqrt{59.45} = \sqrt{\frac{5945}{100}} = \sqrt{\frac{237}{4}}$

$= \frac{1}{2}\sqrt{237}$

$= \frac{1}{2}\left(\sqrt{3} \times \sqrt{79}\right)$

$= \frac{1}{2} \times (1.732 \times 8.888)$

$= \frac{1}{2} \times 15.394$

$= 7.697$

Hence, $\sqrt{59.45} = 7.61$

15. $\sqrt{83.45} = \sqrt{\frac{8345}{100}}$

$= \sqrt{\frac{333}{4}}$

$= \frac{\sqrt{3} \times \sqrt{111}}{\sqrt{4}}$

$= \frac{1.732 \times 10.536}{2}$

$\left(\because \sqrt{3} = 1.732, \sqrt{111} = 10.536\right)$

Hence, $\sqrt{83.45} = 9.124$

MULTIPLE CHOICE QUESTIONS

1. $\sqrt{\overline{39}\,\overline{06}\,\overline{25}}$, square root will have 3 digits.

Hence, option (a) is correct.

2. We have, between n^2 and $(n + 1)^2$, there are $2n$ natural numbers. So, between $(11)^2$ and $(12)^2$ there are 2 × 11 = 22 natural numbers.

Hence, option (c) is correct.

3. $(11111)^2 = 11111 \times 11111$

$= 123454321$

Hence, option (a) is correct.

4. $\sqrt{1.6} \times \sqrt{1.6} = \sqrt{1.6 \times 1.6}$

$= 1.6$

Hence, option (b) is correct.

5. $\sqrt{0.8} \times \sqrt{1.8} = \sqrt{0.8 \times 1.8} = \sqrt{1.44}$

$= 1.2$

Hence, option (c) correct.

6.

$$\begin{array}{r|l} & \;\;1\;4 \\ \hline 1 & \overline{02}\;\overline{02} \\ & \;1 \\ \hline 24 & \;1\;02 \\ 4 & \;\;\;96 \\ \hline & \;\;\;\;\;6 \end{array}$$

$(14)^2 = 196$ & $(15)^2 = 225$

$206 > (14)^2$ by 6.

So, $202 - (14)^2 = 202 - 196 = 6$

Thus, the number 6 must be subtracted from 202 to make it a perfect square.

Hence, option (a) is correct.

7. $\sqrt{0.9} = \sqrt{\frac{9}{10}} = \frac{\sqrt{9}}{\sqrt{10}} = \frac{3}{3.162}$

(from table $\sqrt{10} = 3.152$)

$= 0.94$

Hence, option (b) is correct.

8.

$$\begin{array}{r|l} & \;\;2\;1 \\ \hline 2 & \overline{04}\;\overline{35} \\ & \;4 \\ \hline 41 & \;\;\;35 \\ & \;\;\;41 \\ \hline & \end{array}$$

Since, $(20)^2 = 480$ and $(21)^2 = 441$

So, $(20)^2 < 435 < (21)^2$

$\therefore$ $(21)^2 - 435 = 441 - 435 = 6$

Thus, the number to be added is 6.

Hence, option (d) is correct.

9. (7, 9, 11)

$(11)^2 = 121$

$7^2 + 9^2 = 49 + 81 = 130$

$\therefore$ $(11)^2 < 7^2 + 9^2$

(8, 15, 17)

$(17)^2 = 289$

$8^2 + 15^2 = 64 + 225 = 289$

$(17)^2 = 8^2 + 15^2$, (It is a Pythagorean triplet)

(17, 21, 29)

$(29)^2 = 841$

$(17)^2 + (21)^2 = 289 + 441$

$= 730$

$\therefore$ $(29)^2 > (17)^2 + (21)^2$

And (2, 3, 5)

$5^2 = 25$

$2^2 + 3^2 = 4 + 9 = 13$

$\therefore$ $5^2 > 2^2 + 3^2$

Hence, option (b) is correct.

10. We have the numbers ends with 1, 9, 4, 5, 6 are perfect squares. So, 441 is a perfect square.

Hence, option (b) is correct.

11. $1875 = 3 \times (5 \times 5) \times (5 \times 5)$

3	1875
5	625
5	125
5	25
5	5
	1

There are the number 3 is left unpaired

So, the given number should be divided by 3 to make it perfect square.

Hence, option (c) is correct.

12. In the following number 6084 end with digit 4 (even). So its square ends with even digit.

Hence, option (b) is correct.

13. If a number has 1 or 9 in the unit place, then its square ends with 1.

$\therefore$ $(61)^2$ and $(109)^2$ end with unit digit 1.

Hence option (b) and (d) is correct.

14. $72 = 2 \times (2 \times 2) \times (3 \times 3)$

2	72
2	36
2	18
3	9
3	3
	1

The number 2 is left unpaired.

$\therefore$ The given number must be multiplied by 2 obtain a perfect square.

15. Area of a square field = 196 sq.m

2	196
2	98
7	49
7	7
	1

$\therefore$ Area of square field = $(\text{side})^2$

$\Rightarrow$ $(\text{side})^2 = 196$ sq.m

$\Rightarrow$ side $= \sqrt{196}$m

$= \sqrt{\underline{2 \times 2} \times \underline{7 \times 7}}$

side = 14 m

Hence, option (d) is correct.

MENTAL MATHS CORNER

A. True or False:

1. The square of a prime number is prime. (False)
2. There is no square number between 50 and 60. (True)
3. All square numbers are positive. (True)
4. The product of two square numbers is a square number. (True)
5. The difference between two square numbers is a square number. (False)
6. The sum of two square numbers is a square number. (False)
7. A number ending with even number of zeros is always a perfect square. (False)
8. The square of a natural number is either a multiple of 3 or exceeds. The multiple of 3 by 1. (True)

B. Fill in the blanks:

1. If $\sqrt{x} = y$; then $y^2 = \underline{x}$.
2. Upto 100, there are only **10** numbers which are perfect squares.
3. A rational number whose square is $\frac{81}{64}$ is $\mathbf{\frac{9}{8}}$

   $\therefore$ $\sqrt{\frac{81}{84}} = \sqrt{\frac{9 \times 9}{8 \times 8}} = \frac{9}{8}$
4. The number 57 and 246 when divided by 4 leave the remainder **1** and **2** respectively.
5. The sum of first 19 odd natural numbers is **361.**

   $\therefore$ $1 + 3 + 5 + \ldots\ldots$ 19 times $= (19)^2 = 361$

   $\therefore$ $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25 + 27 + 29 + 31 + 33 + 35 + 37 = 361$

6. A number ending with an odd number of zeros is **never** a perfect square.
7. **Negative** numbers have no square root in the system of rational numbers.
8. A number ending with **2**, **3**, **7** or **8** is never a perfect square.

Review Exercise

1. (*i*) $5184 = 2\times2\times2\times2\times2\times2\times3\times3\times3\times3$

2	5184
2	2592
2	1296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

$\sqrt{5184} = \sqrt{\underline{2\times2}\times\underline{2\times2}\times\underline{2\times2}\times\underline{3\times3}\times\underline{3\times3}}$

$= 2 \times 2 \times 2 \times 3 \times 3$

$= 72$

$\therefore \sqrt{5184} = 72$

(*ii*) 1521

$1521 = 3 \times 3 \times 13 \times 13$

3	1521
3	507
13	169
13	13
	1

$\sqrt{1521} = \sqrt{\underline{3\times3}\times\underline{13\times13}}$

$= 3 \times 13 = 39$

Hence, $\sqrt{1521} = 39$

(*iii*) $3136 = 2\times2\times2\times2\times2\times2\times7\times7$

2	3136
2	1568
2	784
2	392
2	196
2	98
7	49
7	7
	1

$\sqrt{3136} = \sqrt{\underline{2\times2}\times\underline{2\times2}\times\underline{2\times2}\times\underline{7\times7}}$

$= 2 \times 2 \times 2 \times 7$

$= 56$

$\sqrt{3136} = 56$

2. 1 + 3 + 5 + 7 + ... + 23

We have $1 + 3 + 5 + ... (2n - 1) = n^2$

$\Rightarrow \quad 2n - 1 = 23$

$2n = 24$

$n = 12$

and $\quad n^2 = (12)^2 = 144$

Hence,

1 + 3 + 5 + 7 + ... + 23 = 144

3. $(26)^2 = 26 \times 26 = 676$

$(10)^2 + (24)^2 = 100 + 576 = 676$

$\therefore \quad (26)^2 = (10)^2 + (24)^2$

Hence, (10, 24, 26) is a Pythagorean triplet.

4. We have between (n^2) and $(n + 1)^2$, there are 2n natural numbers.

(*i*) between 35 and 36, there are 35 × 2 = 70 natural numbers.

(*ii*) between 100 and 101, there are 100 × 2 = 200 natural numbers.

(*iii*) between 80 and 81, there are 80 × 2 = 160 natural numbers.

5. (*i*)

	7 3
7	$\overline{53}\,\overline{29}$
	49
143	4 29
	4 29
	0

$\sqrt{5329} = 73$

(*ii*)

	2 4 1
2	$\overline{05}\,\overline{80}\,\overline{81}$
	4
44	1 80
	1 76
481	4 81
	4 81
	0

$\sqrt{58081} = 241$

(*iii*)

	5 . 1 9 6 1
5	$\overline{27}.\overline{00}\,\overline{00}\,\overline{00}\,\overline{00}$
	25
101	2 00
	1 01
1029	99 00
	92 61
10386	6 39 00
	6 23 16
103921	15 84 00
	10 39 21
	5 44 79

$\therefore \quad \sqrt{27} = 5.1961$

Hence, $\sqrt{27} = 5.196$

6.

	92
9	$\overline{84}\,\overline{00}$
	81
182	3 00
	3 64

$\therefore$ $(91)^2 = 8281$ & $(92)^2 = 8464$

$(91)^2 < 8400 < (92)^2$

$\therefore$ $(92)^2 - 8400 = 8464 - 8400 = 64$

Thus, the number 64 must be added to given number to make a perfect square.

Now, $8400 + 64 = 8464$

	92
9	$\overline{84}\,\overline{64}$
	81
182	3 64
	3 64
	0

$\sqrt{8464} = 92$

7. (*i*) $\dfrac{(105)^2-(104)^2}{(6)^2-(5)^2} = \dfrac{11025-10816}{36-25} = \dfrac{209}{9}$

(*ii*) $(2881)^2 - (2880)^2 = 8300161 - 8294400$

$= 5761$

8. Let the number of chairs in a row be x. Then

$x \times x = 3025$

$\Rightarrow$ $x^2 = 3025$

$\Rightarrow$ $x = \sqrt{3025} = 55$

Hence, there are 55 chairs in a row.

	5 5
5	$\overline{30}\,\overline{25}$
	25
105	5 25
	5 25
	0

9. The least number which is exactly divisible by each of the numbers 8, 12, 15 and 20 is their L.C.M.

L.C.M. of 8, 12, 15 and 20

$= 2 \times 2 \times 2 \times 3 \times 5 = 120$

2	8,12,15,20
2	4, 6,15,10
3	2, 3, 15, 5
5	2, 1, 5, 5
	2, 1, 1, 1

To make it least square number be multiplied in number 120 by $2 \times 3 \times 5$ *i.e.* 30

So, the required least number $= 120 \times 30 = 3600$

10. Let the number be x. Then

$\dfrac{x}{3} \times \dfrac{x}{9} = 108$

	1 0 8
1	$\overline{01}\,\overline{16}\,\overline{64}$
	1
208	16 64
	16 64
	0

$\Rightarrow$ $\dfrac{x^2}{27} = 108 \Rightarrow x^2 = 108 \times 27 = 2916$

$x = \sqrt{2916} = 54$

Hence, the required number is 54.

HOTS QUESTIONS

1. The largest 4 digit number is 9999.

The square root of 9999.

	9 9
9	$\overline{99}\,\overline{99}$
	81
189	18 99
	17 01
	1 98

$\because$ $(99)^2 = 9801$,

$\therefore$ $99^2 < 9999$ by 198, in order to 4 digit largest square number. Subtracting 198 from 9999.

Thus the required number is

$9999 - 198 = 9801$

2. Let the number be x, Then

$\dfrac{x}{4} \times \dfrac{x}{6} = 486$

$\Rightarrow$ $\dfrac{x^2}{24} = 486$

$\Rightarrow$ $x^2 = 486 \times 24 = 11664$

$x = \sqrt{11664} = 108$

	5 4
5	$\overline{29}\,\overline{16}$
	25
104	4 16
	4 16
	0

Hence, the required number is 108.

3. Let $2m = 18$

$\Rightarrow$ $m = 9$

$\therefore$ $m^2 - 1 = (9)^2 - 1 = 81 - 1 = 80$

and $m^2 + 1 = (9)^2 + 1 = 81 + 1 = 82$

Thus, the required Pythagorean triplet is (18, 80, 82).

Puzzle

Illustration for first ten lockers and first ten students is given below :

Students

Locker Number	1	2	3	4	5	6	7	8	9	10
1	O									
2	O	C								
3	O		C							
4	O	C		O						
5	O				C					
6	O	C	O			C				
7	O						C			
8	O	C		O				C		
9	O		C						O	
10	O	C			O					C

O → Opening the locker

C → Closing the locker

If a number is a perfect square, it will have an odd number of factors

e.g., 9 has three (odd) factors 1, 3 and 9.

So, locker number 9 will be visited by 3 students (1st, 3rd, 9th). First student will open the locker, third will close it and ninth will open it again. (*see* illustration)

Whereas if a number is a non-perfect square, it will have an even number of factors

e.g., 8 has four (even) factors 1, 2, 4 and 8.

So, locker number 8 will be visited by 4 students (1st, 2nd, 4th, 8th). First student will open the locker, second will close it, fourth student will open it and eighth will close it. (*see* illustration)

We see that, if a particular locker is visited an odd number of times (in case of perfect squares) it will be open at the end of the procedure, otherwise it will be closed (in case of non-perfect squares).

So, the open lockers are numbered 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, all of which are perfect squares.

Total number of lockers opened is 10.