Std. 12th

MATHS

PRACTICAL HANDBOOK

Instruction for Students :

1. L.H.S. means Left Hand Side (Blank page of practical record) and R.H.S. means Right Hand Side (line page of practical record)

2. L.H.S. page of each and every experiment should be written by pencil only.

3. R.H.S. page of each and every experiment should be written by blue/black pen.

4. Diagrams should be drawn neatly and should be properly labelled.

5. Graphs will be drawn on separate graph paper after noting observations on performing experiment.

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Experiment No. 1

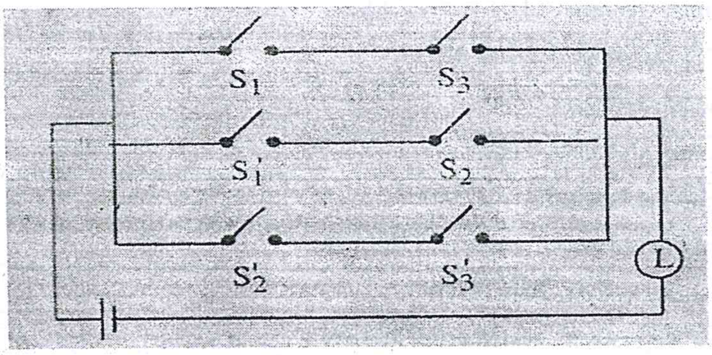
Aim : Applications of logic

Problem 1 : Express the following switching circuit in the symbolic form. Also construct its switching table and identify the state of switches when lamp is off.

Formula :

p	q	$p \vee q$	$p \wedge q$
1	1	1	1
1	0	1	0
0	1	1	0
0	0	0	0

Figure :



Observation Table :

The switching table for the given switching circuit as follows :

Switches			Sub-symbolic forms							Lamp l
P	Q	R	$\sim p$	$\sim q$	$\sim r$	$(p \wedge r)$	$\sim p \wedge q$	$(p \wedge r) \vee (\sim p \vee q)$	$(\sim q \wedge \sim r)$	
1	1	1	0	0	0	1	0	1	0	1
1	1	0	0	0	1	0	0	0	0	0
1	0	1	0	1	0	1	0	1	0	1
1	0	0	0	1	1	0	0	0	1	1
0	1	1	1	0	0	0	1	1	0	1
0	1	0	1	0	1	0	1	1	0	1
0	0	1	1	1	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0	1	1

Conclusion :

i. Symbolic form $\iota \equiv [p \wedge r] \vee [\sim p \wedge q] \vee [\sim q \wedge \sim r]$

ii. The lamp l is off in two cases

a) S_1 and S_2 are on and S_3 is off

b) S_1 and S_2 are off and S_3 is on.

Experiment No. 1

Aim : Applications of logicType equation here.

Problem 1 : Express the following switching circuit in the symbolic form. Also construct its switching table and identify the state of switches when lamp is off.

Solution : Let
p : switch S_1 is closed.
q : switch S_2 is closed
r : switch S_3 is closed
l : the lamp L is on.
$\sim p$:switch S1 closed
$\sim q=$ switch S2 is closed
$\sim r=$ switch S3 is closed

Therefore the symbolic form of logic of the given circuit is

$$l = [p \wedge r] \vee [\sim p \wedge q] \vee [\sim q \wedge \sim r]$$

From the switching table, we observe that the lamp L is off, in two cases, when states of switches.

i. S_1 and S_2 are on and S_3 is off,
ii. S_1 and S_2 are off and S_3 is on and in other cases lamp is on.

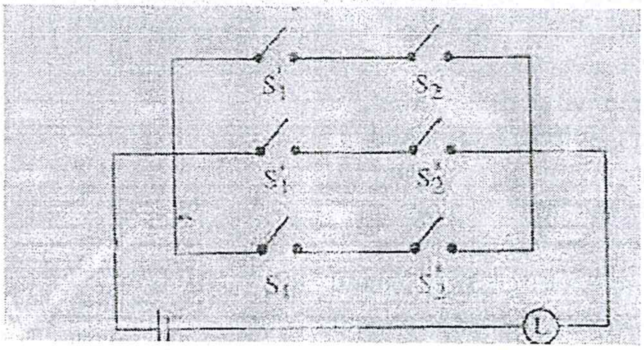
Conclusion :

i. The symbolic form $l = [p \wedge r] \vee [\sim p \wedge q] \vee [\sim q \wedge \sim r]$
ii. The lamp L is OFF in two cases.
a) S_1 and S_2 are on and S_3 is OFF.
b) S_1 and S_2 are off and S_3 is ON.

Experiment No. 1

Problem 2 : Construct the simplified form of the following circuit with two switches only.

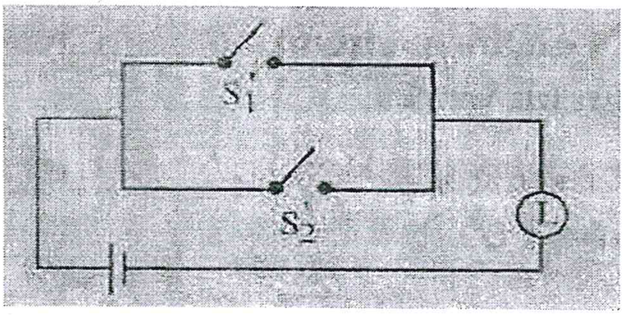
Diagram :



Formula :

1. Distributive Law
   a) $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$
   b) $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$
2. Complement Law
   a) $p \vee \sim p \equiv T$
   b) $P \wedge \sim P \equiv F$
3. Identity Law
   a) $P \vee F \equiv P$
   b) $P \wedge T \equiv P$

Conclusion : The simplified form of the given circuit is
$l \equiv \sim p \vee \sim q$ &
The switching circuit is



Experiment No. 1

Problem 2 : Construct the simplified form of the following circuit with two switches only.

Solution : Let p : switch S_1 is closed
q : switch S_2 is closed
l : lamp L is ON
$\sim p$: switch S_1 is closed.
$\sim q$: switch S_2 is closed.

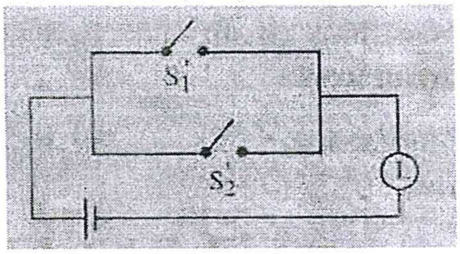
Therefore the symbolic form of switching circuit is

$$l \equiv [\sim p \wedge q] \vee [\sim p \wedge \sim q] \vee [p \wedge \sim q]$$

Now on simplification we have,

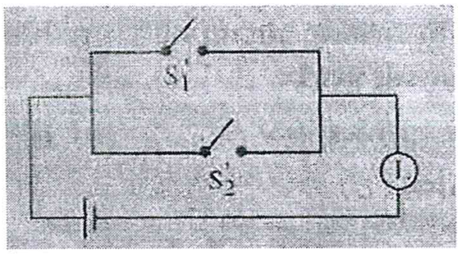
$$l \equiv [\sim p \wedge q] \vee [\sim p \wedge \sim q] \vee [p \wedge \sim q]$$
$$\equiv [\sim p \wedge q] \vee [(\sim p \vee p) \wedge \sim q] \quad \text{(by distributive law.)}$$
$$\equiv [\sim p \wedge q] \vee [T \wedge \sim q] \quad \text{(by compliment law)}$$
$$\equiv [\sim p \wedge q] \vee [\sim q] \quad \text{(by identity law)}$$
$$\equiv [\sim p \vee \sim q] \wedge [q \vee \sim q] \quad \text{(by distributive law)}$$
$$\equiv [\sim p \vee \sim q] \wedge T \quad \text{(by complement law)}$$
$$\equiv [\sim p \vee \sim q] \quad \text{(by identity law)}$$

The simplified circuit of $l \equiv (\sim p \vee \sim q)$ is adjacent figure.



Conclusion :

The simplified form of the given circuit is
$l \equiv (\sim p \vee \sim q)$ & the switching circuit is



Experiment No. 2

LHS

Aim : Inverse of a matrix by Adjoint method and hence solution of system of linear equations.

Problem 1 : Find the inverse of a matrix by Adjoint method $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

Formula :

Step I : Verify that A is a square matrix.

Step II : Compute $|A|$ and verify that $|A| \neq 0$

Step III : Find the cofactors of $A_{ij} = (-1)^{i+j} M_{ij}$ of each element a_{ij} of matrix A and form the matrix of cofactors $[A_{ij}]$

Step IV : Find the transpose of the matrix$[A_{ij}]$ and express adj A= $[A_{ij}]$

Step V : $A^{-1} = \frac{Adj\,[A]}{|A|}$

Conclusion : $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$ is found by Adjoint method

Experiment No. 2

Aim : Inverse of a matrix by Adjoint method and hence solution of system of linear equations.

Problem 1 : Find the inverse of a matrix by adjoint method $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

Solution :

Step I : The given matrix is a square matrix of order 3.

Step II : Let A= $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ $\therefore$ $|A| = \begin{vmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{vmatrix}$

$|A| = 3(1) - (-3)(2) + 4(-2)$

$= 3+6-8$

$|A| = 1 \neq 0$

$\therefore$ A^{-1} exists

Step III : $A_{11} = (-1)^{1+1}(-3+4) = 1$; $A_{12} = (-1)^{1+2}(2-0) = -2$

$A_{13} = (-1)^{1+3}(-2-0) = -2$; $A_{21} = (-1)^{2+1}(-3+4) = -1$

$A_{22} = (-1)^{2+2}(3-0) = 3$; $A_{23} = (-1)^{2+3}(-3-0) = 3$

$A_{31} = (-1)^{3+1}(-12+12) = 0$; $A_{32} = (-1)^{3+2}(12-8) = -4$

$A_{33} = (-1)^{3+3}(-9+6) = -3$

The matrix of cofactors

$$[A_{ij}] = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{bmatrix}$$

Step IV : $\text{Adj}(A) = [A_{ij}]^T = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

Step V : $A^{-1} = \frac{Adj[A]}{|A|} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

Conclusion : $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$ of a matrix is found by Adjoint method

Experiment No. 2

Problem 2 : Transform the linear equations $x + 2y + 3z = 9; 2x + 3y + z = 4; 4x + 5y + 4z = 15$ in a matrix equation and solve them by finding inverse of coefficient matrix using adjoint method.

Formula :

Step I : Transform the simultaneous linear equation into a matrix equation A x = B

Step II : Separate the matrix of coefficients A, variables X and constants B from matrix equation

Step III : Using one of the methods of inverse, find A^{-1}.

Step IV : Compute the product $A^{-1}B$

Step V : Express X= $A^{-1}B$ and convert the matrix equation into linear equations, which gives the value of variable x,y, z.

Conclusion : By Adjoint method the solution of given linear equation is x=2, y= -1 and z=3

Experiment No. 2

Problem 2: Transform the linear equations $x + 2y + 3z = 9, 2x + 3y + z = 4, 4x + 5y + 4z = 15$ in a matrix equation and solve them by finding inverse of co-efficient matrix using adjoint method.

Solution :

Step I : The matrix equation of given simultaneous linear equation is

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 4 & 5 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ 15 \end{bmatrix}$$

Step II: Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 4 & 5 & 4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 9 \\ 4 \\ 15 \end{bmatrix}$ be the matrices of coefficients, variables and constants respectively.

Step III :The matrix A is a square matrix of order 3.

Now $|A| = 1(12 - 5) - 2(8 - 4) + 3(10 - 12)$

$= 7 - 8 - 6$

$|A| = -7 \neq 0$,

Hence A^{-1} exists

We have, $A_{11} = 7, A_{12} = -4, A_{13} = 2,$

$A_{21} = 7, A_{22} = -8, A_{23} = 3$

$A_{31} = -7, A_{22} = 5, A_{33} = -1$

$$\because [A_{ij}] = \begin{bmatrix} 7 & -4 & -2 \\ 7 & -8 & 3 \\ -7 & 5 & -1 \end{bmatrix} \quad \therefore adj(A) = [A_{ij}]^T = \begin{bmatrix} 7 & 7 & -7 \\ -4 & -8 & 5 \\ -2 & 3 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} Adj\,(A) = \quad \therefore A^{-1} = \frac{-1}{7}\begin{bmatrix} 7 & 7 & -7 \\ -4 & -8 & 5 \\ -2 & 3 & -1 \end{bmatrix}$$

$$A^{-1}B = \frac{-1}{7}\begin{vmatrix} 7 & 7 & -7 \\ -4 & -8 & -5 \\ -2 & 3 & -1 \end{vmatrix}\begin{vmatrix} 9 \\ 4 \\ 15 \end{vmatrix}$$

$$= \frac{-1}{7}\begin{bmatrix} 63 + 28 - 105 \\ -36 - 32 + 15 \\ -18 + 12 - 15 \end{bmatrix} = -\frac{1}{7}\begin{bmatrix} -14 \\ 7 \\ -21 \end{bmatrix} = \begin{vmatrix} 2 \\ -1 \\ 3 \end{vmatrix}$$

Step V: $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

Compare the matrix $x = 2, y = -1, z = 3$

Conclusion : By adjoint method the solution of given liner equations is x=2, y=-1 and 2= 3

Experiment No. 3 LHS

Aim : Inverse of a matrix by Elementary Transformation and hence solution of system of linear equations.

Problem 1 : Find the inverse of matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$

Formula :

Step I : Verify that A is a square matrix.

Step II : Verify that

$|A| \neq 0$

Step III : Consider the unit matrix I of the order of matrix A and write the matrix equation as

a) $AA^{-1} = I$ for solving by row transformations

b) $A^{-1}A = I$ for solving by column transformations.

Step IV : One by one select suitable row in (a) and column in (b) transformation and perform them on prefactor A in (a) and postfactor A in (b) of left side and on I of right side of matrix equation, so that A reduces to I and I changes to say matrix B

$\therefore$ (a) $IA^{-1} = B$ or (b) $A^{-1}I = B$

To use minimum number of transformations reduce first diagonal element 1 and them non diagonal element to 0, column wise in (a) and rowwise in (b) of matrix A.

Step V : Result $A^{-1} = B$

Conclusion : $A^{-1} = \frac{1}{9}\begin{bmatrix} 0 & 3 & 3 \\ 3 & 2 & -1 \\ 3 & -7 & -1 \end{bmatrix}$ is a matrix found by Elementary method.

Experiment No. 3

Aim : Inverse of a matrix by Elementary Transformation and hence solution of system of linear equations.

Problem 1 : find the inverse of matrix $A = \begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{vmatrix}$ by elementary column operations.

Solution :

Step I : A is a square matrix of order 3

Step II : Consider $|A| = \begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{vmatrix}$

$\therefore\ |A| = 1(1-1) - 2(0+3) + 1(0-3) = 0 - 6 - 3$

$|A| = -9 \neq 0$

Therefore A is a non-singular square matrix and hence A^{-1} exists.

Step III : Let $A^{-1} A = I$

$$\therefore A^{-1} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step IV : Operate $C_2 \to C_2 - 2C_1$ and $C_3 \to C_3 - C_1$

$$A^{-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 3 & -7 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Operate : $C_3 \to C_3 + C_2$

$$A^{-1} \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & -7 & -9 \end{vmatrix} = \begin{vmatrix} 1 & -2 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

Operate : $C_3 \to -\frac{1}{9} C_3$

$$A^{-1} \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & -7 & 1 \end{vmatrix} = \begin{vmatrix} 1 & -2 & \frac{1}{3} \\ 0 & 1 & \frac{-1}{9} \\ 0 & 0 & \frac{-1}{9} \end{vmatrix}$$

Operate : $C_1 \to C_1 - 3C_3,\ C_2 \to C_2 + 7C_3$

$$A^{-1} \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{9} & \frac{-1}{9} \\ \frac{1}{3} & \frac{-7}{9} & \frac{-1}{9} \end{vmatrix}$$

Step V : $A^{-1} = \frac{1}{9} = \begin{bmatrix} 0 & 3 & 3 \\ 3 & 2 & -1 \\ 3 & -7 & -1 \end{bmatrix}$

Conclusion : $A^{-1} = \frac{1}{9} \begin{bmatrix} 0 & 3 & 3 \\ 3 & 2 & -1 \\ 3 & -7 & -1 \end{bmatrix}$ is a matrix found by Elementary method.

Experiment No. 3

Problem 2 : Express the linear equations

$$x + 2y + 3z = 1$$
$$2x + 5y + 6z = 2$$
$$3x + 7y + 8z = 4$$

into a matrix equation. Also find the inverse by elementary transformations of co-efficient matrix and solve the equations.

Formula :

Step I : Transform the simultaneous linear equation into a matrix equation AX = B

Step II : Separate the matrix of coefficient A_1 variable x and constant B from the matrix equation

Step III : Using one of the method inverse find A^{-1}.

Step III : Compute the product AB

Step IV : Express X= A^{-1}B and convert matrix equation into linear equation which gives the value of variable.

Conclusion : By elementary transformation method the solution of given linear equation is x=4, y=0, and z= -1 is the required solution.

Problem 2 : Express the linear equations

$x + 2y + 3z = 1$

$2x + 5y + 6z = 2$

$3x + 7y + 8z = 4$ into a matrix equation.

Also find the inverse by elementary transformations of coefficient matrix and solve the equations.

Solution :

Step I : The matrix equation of the given simultaneous linear equation is

$$\begin{bmatrix}1&2&3\\2&5&6\\3&7&8\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} \quad \text{and} \quad B=\begin{bmatrix}1\\2\\4\end{bmatrix}$$

Step II : Let $A=\begin{bmatrix}1&2&3\\2&5&6\\3&7&8\end{bmatrix}$, $x=\begin{bmatrix}x\\y\\z\end{bmatrix}$ and $B=\begin{bmatrix}1\\2\\4\end{bmatrix}$ be the matrices of coefficients, variables and constants respectively.

Step III : Let $AA^{-1} = I$

$$\begin{bmatrix}1&2&3\\2&5&6\\3&7&8\end{bmatrix}A^{-1}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

Performing $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 3R_1$, we have

$$\begin{vmatrix}1&2&3\\0&1&0\\0&1&-1\end{vmatrix}A^{-1}=\begin{vmatrix}1&0&0\\-2&1&0\\-3&0&1\end{vmatrix}$$

Performing $R_1 \to R_1 - 2R_2$ and $R_3 \to R_3 - R_2$ we get

$$\begin{vmatrix}1&0&3\\0&1&0\\0&0&-1\end{vmatrix}A^{-1}=\begin{vmatrix}5&-2&0\\-2&1&0\\-1&-1&1\end{vmatrix}$$

Performing $R_1 \to R_1 + 3R_3$

$$\begin{vmatrix}1&0&0\\0&1&0\\0&0&-1\end{vmatrix}A^{-1}=\begin{vmatrix}2&-5&3\\-2&1&0\\-1&-1&1\end{vmatrix}$$

Performing $R_3 \to -R_3$

$$\begin{vmatrix}1&0&0\\0&1&0\\0&0&1\end{vmatrix}A^{-1}=\begin{vmatrix}2&-5&3\\-2&1&0\\1&1&-1\end{vmatrix}$$

$$A^{-1}=\begin{vmatrix}2&-5&3\\-2&1&0\\1&1&-1\end{vmatrix}$$

Step IV : $$A^{-1}B=\begin{vmatrix}2&-5&3\\-2&1&0\\1&1&-1\end{vmatrix}\begin{vmatrix}1\\2\\4\end{vmatrix}=\begin{vmatrix}2-10+12\\-2+2+0\\1+2-4\end{vmatrix}=\begin{vmatrix}4\\0\\-1\end{vmatrix}$$

Step V : $$\begin{vmatrix}x\\y\\z\end{vmatrix}=\begin{vmatrix}4\\0\\-1\end{vmatrix}$$ Compare the matrix $x = 4, y = 0, z = -1$

Conclusion : By elementary transformation method the solution of given equation is $x = 4, y = 0, and\ z = -1$ is the required solution.

Experiment No. 4

Aim : Solution of a triangle.

Problem 1: In a $\Delta ABC, Sin\ (2A - B) = \frac{1}{2}$ and angles A,B and C are in A. P Determine the values of angles A, B and C.

Formula : The sum of angles of triangles are 180°

$A + B + C = \pi$

$\sin 30° = \frac{1}{2}$

Conclusion : The values of angles A= 45°, B= 60°, C= 75°

Experiment No. 4

Aim : Solution of a triangle.

Problem 1 : In a $\Delta ABC, Sin\ (2A - B) = \frac{1}{2}$ and angles A,B and C are in A. P. Determine the values of angles A, B and C.

Solution : Since the angles A, B and C of the triangle are in A.P.

$\therefore A + B + C = \pi$ and $2B = A + B$

$\therefore 3B = \pi$ Hence $B = \frac{\pi}{3} = 60°$ and

$A + C = 120°$

Since $\sin(2A - B) = \frac{1}{2}$ $\therefore 2A - B = 30°$

$\therefore 2A = 60° + 30° = 90°$

$\therefore A = 45°$

$\therefore C = 120° - 45°$

$\therefore A = 45°, B = 60°, C = 75°$

Conclusion : The values of angles A= 45°, B= 60°, C= 75°

Experiment No. 4

Problem 2 : If A+B+C=180°, prove that

$$\sin\left(\frac{A}{2}\right) + \sin\left(\frac{B}{2}\right) + \sin\left(\frac{C}{2}\right) = 1 + 4\sin\left(\frac{\pi-A}{4}\right)\sin\left(\frac{\pi-B}{4}\right)\sin\left(\frac{\pi-C}{4}\right)$$

Formula:

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$A + B + C = \pi$$

$$\cos(\pi - C) = 1 - 2sin^2\left(\frac{\pi-C}{2}\right)$$

$$\cos A - \cos B = 2\sin\left(\frac{A-B}{2}\right)\sin\left(\frac{A+B}{2}\right)$$

Conclusion : Hence it is proved that

$$\sin\left(\frac{A}{2}\right) + \sin\left(\frac{B}{2}\right) + \sin\left(\frac{C}{2}\right) = 1 + 4\sin\left(\frac{\pi-A}{4}\right)\sin\left(\frac{\pi-B}{4}\right)\sin\left(\frac{\pi-C}{4}\right)$$

Experiment No. 4

Problem 2 : If A+B+C=180°, prove that

$$\sin\left(\frac{A}{2}\right) + \sin\left(\frac{B}{2}\right) + \sin\left(\frac{C}{2}\right) = 1 + 4\sin\left(\frac{\pi-A}{4}\right)\sin\left(\frac{\pi-B}{4}\right)\sin\left(\frac{\pi-C}{4}\right)$$

Solution :

$$\begin{aligned}
\text{LHS} &= \left|\sin\left(\frac{A}{2}\right) + \sin\left(\frac{B}{2}\right)\right| + \left|\sin\left(\frac{C}{2}\right)\right| \\
&= 2\sin\left(\frac{A+B}{4}\right)\cos\left(\frac{A-B}{4}\right) + \cos\left(\frac{\pi}{2} - \frac{C}{2}\right) \\
&= 2\sin\left(\frac{\pi-C}{4}\right)\cos\left(\frac{A-B}{4}\right) + 1 - 2\sin^2\left(\frac{\pi-c}{4}\right) \\
&= 1+2\sin\left(\frac{\pi-c}{4}\right)\left|\cos\left(\frac{A-B}{4}\right) - \sin\left(\frac{\pi-C}{4}\right)\right| \\
&= 1+2\sin\left(\frac{\pi-c}{4}\right)\left|\cos\left(\frac{A-B}{4}\right) - \sin\left(\frac{A+B}{4}\right)\right| \\
&= 1+2\sin\left(\frac{\pi-c}{4}\right)\left|\cos\left(\frac{A-B}{4}\right) - \cos\left(\frac{\pi}{2} - \frac{A+B}{4}\right)\right| \\
&= 1+2\sin\left(\frac{\pi-c}{4}\right)\left|2\sin\left(\frac{\pi-B}{4}\right)\sin\left(\frac{\pi-A}{4}\right)\right| \\
&= 1+4\sin\left(\frac{\pi-A}{4}\right)\sin\left(\frac{\pi-B}{4}\right)\sin\left(\frac{\pi-C}{4}\right)
\end{aligned}$$

Conclusion : Hence it is proved that

$$\sin\left(\frac{A}{2}\right) + \sin\left(\frac{B}{2}\right) + \sin\left(\frac{C}{2}\right) = 1 + 4\sin\left(\frac{\pi-A}{4}\right)\sin\left(\frac{\pi-B}{4}\right)\sin\left(\frac{\pi-C}{4}\right)$$

Experiment No. 5

LHS

Aim : Inverse trigonometric functions.

Problem 1: Prove that $\sin^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \sin^{-1}\left(\frac{56}{65}\right)$

Formula : $\cos^2\theta = 1 - \sin^2\theta$

$\cos^2\theta + \sin^2\theta = 1$

$\sin(\theta_1 + \theta_2) = \sin\theta_1\cos\theta_2 + \cos\theta_1$

Conclusion : Hence proved that

$\sin^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \sin^{-1}\left(\frac{56}{65}\right)$

Experiment No. 5

Aim : Inverse trigonometric functions.

Problem 1 : Prove that $\sin^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \sin^{-1}\left(\frac{56}{65}\right)$

Solution : Let $\sin^{-1}\left(\frac{3}{5}\right) = \theta_1$, $\sin(\theta_1) = \frac{3}{5}$ and $0 < \theta_1 < \frac{\pi}{2}$

$\cos^2\theta_1 = 1 - \sin^2\theta_1$

$\therefore \cos^2\theta_1 = 1 - \frac{9}{25} = \frac{16}{25}$

Hence $\cos\theta_1 = \frac{4}{5}$ as $0<\theta_1<\frac{\pi}{2}$

Let $\cos^{-1}\frac{12}{13} = \theta_2$ $\quad\therefore \cos\theta_2 = \frac{12}{13}$ and as $0<\theta_2<\frac{\pi}{2}$

Since $\cos^2\theta_2 + \sin^2\theta_2 = 1$ $\quad\therefore \sin^2\theta_2 = 1 - \frac{144}{169} = \frac{25}{169}$

$\therefore \sin\theta_2 = \frac{5}{13}$ as $0 < \theta_2 < \frac{\pi}{2}$

$\because \sin(\theta_1 + \theta_2) = \sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2$

$= \frac{3}{5} \times \frac{12}{13} + \frac{4}{5} \times \frac{5}{13} = \frac{56}{65}$

$\sin(\theta_1 + \theta_2) = \frac{56}{65}$

$\therefore \theta_1 + \theta_2 = \sin^{-1}\left(\frac{56}{65}\right)$

i.e. $\sin^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \sin^{-1}\left(\frac{56}{65}\right)$

Conclusion : Hence proved that

$$\sin^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \sin^{-1}\left(\frac{56}{65}\right)$$

Experiment No. 5

Problem 2: Solve this equation $\sin^{-1} x + \sin^{-1} 2x = \frac{\pi}{3}$

Formula : $\sin^2 x + \cos^2 x = 1$

$\cos(\theta_1 + \theta_2) = \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2$

Conclusion : Solution of equation

$\sin^{-1} x + \sin^{-1} 2x = \frac{\pi}{3}$ is $x = \pm\sqrt{\frac{3}{28}}$

Experiment No. 5

Problem 2 : Solve the equation $\sin^{-1} x + \sin^{-1} 2x = \frac{\pi}{3}$

Solution : The given equation is $\sin^{-1} x + \sin^{-1} 2x = \frac{\pi}{3}$

Let $\sin^{-1} x = \theta_1$ and $\sin^{-1} 2x = \theta_2$

$\therefore \sin\theta_1 = x$ and $\sin\theta_2 = 2x$

$\therefore \cos\theta_1 = \sqrt{1-x^2}$ and $\cos\theta_2 = \sqrt{1-4x^2}$

Also, $\theta_1 + \theta_2 = \frac{\pi}{3}$ $\quad \therefore \cos(\theta_1+\theta_2) = \cos\frac{\pi}{3}$

$\cos\theta_1 \cos\theta_2 - \sin\theta_1 \cos\theta_2 = \frac{1}{2}$

i.e. $\sqrt{1-x^2}\sqrt{1-4x^2} - x \times 2x = \frac{1}{2}$

i.e. $2\sqrt{1-x^2}\sqrt{1-4x^2} = 1 + 4x^2$

now squaring both sides, we get

$4(1-x^2)(1-4x^2) = (1+4x^2)^2$

$4(1 - x^2 - 4x^2 + 4x^4) = 1 + 8x^2 + 16x^4$

i.e. $28x^2 = 3$ $\quad$ i.e. $x^2 = \frac{3}{28}$

hence $\quad x = \pm\sqrt{\frac{3}{28}}$

Conclusion : solution of equation $\sin^{-1} x + \sin^{-1} 2x = \frac{\pi}{3}$ is $x = \pm\sqrt{\frac{3}{28}}$

Experiment No. 6

Aim : Geometrical Applications of Vectors.

Problem 1 : Prove the perpendicular bisectors of sides of a triangle are concurrent.

Formula : Mid point formula

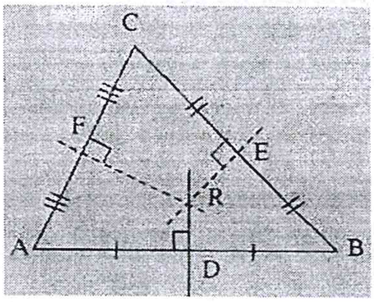
$$x = \frac{m+n}{2}$$

$\therefore$ lines are perpendicular

$RD \perp AB$

$\therefore \overrightarrow{RD}\ \overrightarrow{AB} = 0$

Figure :



Conclusion : Perpendicular bisector RF of side AC also passes through common point of intersection R. hence the perpendicular bisectors of sides of triangle are concurrent.

Experiment No. 6

Problem 1 : Prove the perpendicular bisectors of sides of a triangle are concurrent.

Solution : Let D, E, F be the mid-points of sides AB, BC and CA respectively of ΔABC

$\therefore \vec{d} = \frac{\vec{a}+\vec{b}}{2}, \quad \vec{e} = \frac{\vec{b}+\vec{c}}{2}, \quad$ and $\vec{f} = \frac{\vec{a}+\vec{b}}{2}$

Let R be the point of intersection of perpendicular bisectors of sides AB and BC.

$\therefore$ RD $\perp$ AB and RE $\perp$ BC $= 0$

$\therefore \overrightarrow{RD} \cdot \overrightarrow{AB} = 0$ and $\overrightarrow{RE} \cdot \overrightarrow{BC} = 0$

i.e. $(\vec{d} - \vec{r}) \cdot \overrightarrow{AB} = 0$ and $(\vec{e} - \vec{r}) \cdot \mathrm{BC} = 0$

i.e. $\vec{d} \cdot \overrightarrow{AB} - \vec{r} \,.\, \overrightarrow{AB} = 0$ and $\vec{e} \cdot \overrightarrow{BC} - \vec{r} \cdot \overrightarrow{BC} = 0$

i.e. $\left(\frac{\vec{a}+\vec{b}}{2}\right) . (\vec{b} - \vec{a}) - \vec{r} \cdot \overrightarrow{AB} = 0$ and

$\left(\frac{\vec{b}+\vec{c}}{2}\right) (\vec{c} - \vec{b}) - \vec{r} \cdot \overrightarrow{BC} = 0$

i.e. $\frac{b^2-a^2}{2} - \vec{r} \cdot \overrightarrow{AB} = 0$ and $\frac{c^2-b^2}{2} - \mathrm{r} \cdot \overrightarrow{BC} = 0$

Now adding both equations we get,

$\frac{c^2-a^2}{2} - \vec{r} \cdot (\overrightarrow{AB} + \overrightarrow{BC}) = 0$

$\therefore \left(\frac{\vec{a}+\vec{c}}{2}\right) \cdot (\vec{c} - \vec{a}) - \vec{r} \cdot \overrightarrow{AC} = 0$

i.e. $\vec{f} \cdot \overrightarrow{AC} - \vec{r} \cdot \overrightarrow{AC} = 0$

i.e. $(\vec{f} - \vec{r}) \cdot \overrightarrow{AC} = 0$

i.e. $\overrightarrow{RF} \cdot \overrightarrow{AC} = 0$

Conclusion : Perpendicular bisector RF of side AC also passes through common point of intersection R. hence the perpendicular bisectors of sides of triangle are concurrent.

Experiment No. 6

Problem 2 : Prove that if two pairs of opposite edges of a tetrahedron are orthogonal, then third pair is also orthogonal.

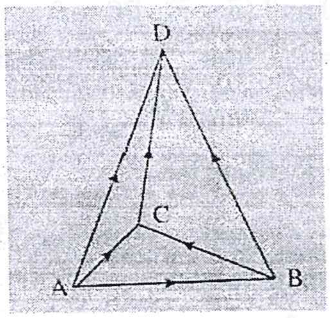
Formula: ABCD be orthogonal

$\overrightarrow{AD} \cdot \overrightarrow{BC} = 0$

$(\vec{d} - \vec{a}) \cdot (\vec{c} - \vec{b}) = 0$

$\therefore \overrightarrow{AB} \cdot \overrightarrow{CD} = 0$

Figure:



Conclusion: The third pair (AB, CD) of opposite edges of a tetrahedron is also orthogonal.

Experiment No. 6

Problem 2 : Prove that if two pairs of opposite edges of a tetrahedron are orthogonal, then third pair is also orthogonal.

Solution : Let the two pairs (AD, BC) and (BD, AC) of opposite edges of tetrahedron ABC.D be orthogonal.

$\therefore\ \overrightarrow{AD}\cdot\overrightarrow{BC} = 0$ and $\overrightarrow{BD}\cdot\overrightarrow{AC} = 0$

$\therefore\ (\vec{d}-\vec{a})\cdot(\vec{c}-\vec{b}) = 0$ and

$(\vec{d}-\vec{b})\cdot(\vec{c}\cdot\vec{a}) = 0$

$\therefore\ \vec{d}\cdot\vec{c}-\vec{d}\cdot\vec{b}-\vec{a}\cdot\vec{c}+\vec{a}\cdot\vec{b} = 0$ and

$\vec{d}\cdot\vec{c}-\vec{d}\cdot\vec{a}-\vec{b}.\vec{c}+\vec{b}\cdot\vec{a} = 0$

$\therefore\ \vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c} = \vec{d}\cdot\vec{a}+\vec{b}\cdot\vec{c}$

i.e. $\vec{d}\cdot(\vec{b}\cdot\vec{a})\cdot(\vec{d}-\vec{c}) = 0$

i.e. $\overrightarrow{AB}\cdot\overrightarrow{CD} = 0$

Conclusion : The third pair (AB, CD) of opposite edges of a tetrahedron is also orthogonal.

Experiment No. 7

Aim : Three Dimensional Geometry (d.r.s. and d.c.s.)

Problem 1 : Find the co-ordinates of foot of the perpendicular drawn from the point $p \equiv (1,2,1)$ to the line joining the point $A \equiv (1,4,6)$ and $B \equiv (5,4,4)$

Formula : $PM \perp AB$

Cartesian form

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} = k$$

$\because AB \parallel AM$

d.c.s. of lines

$$\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$$

Conclusion : The d.c.s. of two lines are

$$\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \text{ and } \frac{-1}{\sqrt{6}} \frac{1}{\sqrt{6}} \frac{2}{\sqrt{6}}$$

Experiment No. 7

Aim : Three Dimensional Geometry (d.r.s. and d.c.s.)

Problem 1: Find the co-ordinates of foot of the perpendicular draw from the point $P \equiv (1,2,1)$ to the line joining the points $A \equiv (1,4,6)$ and $B \equiv (5,4,4)$

Solution: Let $M \equiv (a, b, c)$ be the co-ordinates of the foot of the perpendicular

$\therefore$ d.r.s. of PM are $(a-1, b-2, c-1)$ and of AB are 4,0,-2 i.e. 2,0,-1

$\therefore$ PM is $\perp$ to AB

$\therefore 2(A-1) + 0(B-2) - 1(C-1) = 0$

i.e. 2a-c=1

$\because AB \parallel AM \qquad \therefore let\ \frac{a-1}{2} = \frac{b-4}{0} = \frac{c-6}{-1} = k$

$\therefore \quad a = 2k + 1$ and $c = -k + 6$

$\therefore 2a\text{-}c= 4k+2+k\text{-}6=1$

$i.e. 5k = 5 \qquad i.e.\ k=1$

Hence $\frac{a-1}{2} = \frac{b-4}{0} = \frac{c-6}{-1} = 1$

$\therefore a = 3, b = 4, c = 5 \qquad \therefore M = (3,4,5)$

$\therefore$ the d.r.s of two line are $\frac{1}{3}, n, \frac{2}{3}n, n$ and $-\frac{1}{2}n, \frac{1}{2}n, n$

$i.e.\quad 1,2,3 \quad and \quad -1,1,2$

Hence $\sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$

And $\sqrt{(-1)^2 + 1^2 + 2^2} = \sqrt{6}$

Conclusion: $\therefore$ The d.c.s. of two line are

$\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$ and $\frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}$

Experiment No. 7

Problem 2 : Prove that the two lines whose direction cosines are given by the relations
$al + bm + cm = 0$ and $fmn + hlm + gnl = 0$

Formula : Product of roots

$$\frac{m_1}{n_1} \times \frac{m_2}{n_2} = \frac{cg}{bh}$$

$$\frac{c_1c_2}{f/_a} = \frac{m_1m_2}{g/_b} = \frac{n_1n_2}{h/_c} = k$$

Since the given line are perpendicular

$$\therefore l_2l_2 + m_1m_2 + n_1n_2 = 0$$

Conclusion : Hence proved that two line al+bm+cn=0 and fmn+hnm+nlw=0 are perpendicular if $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$

Experiment No. 7

Problem 2: Prove that the two lines whose direction cosines are given by the relations $al + bm + cn = 0$ and $fmn + gnl + hlm = 0$ are perpendicular if $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$

Solution : Given that $al + bm + cn = 0$ ------ (1)

And $fmn + gnl + hlm = 0$ ------ (2)

Now from (1) $l = \frac{-bm+cn}{a}$ and

From (2) $l = \frac{-fmn}{gn+hm}$ for elimination of l

$$\therefore \frac{-bm + cn}{a} = \frac{-fmn}{gn + hm}$$

$\therefore$ $bmgn + bm^2h + cn^2g + cnhm = afmn$

i.e. $bhm^2 + (bg + lh - af)nm + cgn^2 = 0$

i.e. $bh\left(\frac{m}{n}\right)^2 + (bg + ch - af)\left(\frac{m}{n}\right) + cg = 0$

let $\frac{m_1}{n_1}$ and $\frac{m_2}{n_2}$ be the roots of quadratic equations

$\therefore$ the product of roots $= \frac{m_1}{n_1} \times \frac{m_2}{n_2} = \frac{cg}{bh}$

i.e. $\frac{m_1m_2}{g/b} = \frac{n_1n_2}{h/c}$ (3)

similarly by eliminating m from equation (1) and (2) we will get.

$\frac{l_1l_2}{f/a} = \frac{n_1n_2}{h/c}$ ---------- (4)

$\therefore$ from (3) and (4) we have,

$\frac{l_1l_2}{f/a} = \frac{m_1m_2}{g/b} = \frac{n_1n_2}{h/c} = k$

$\therefore l_1l_2 = k\left(\frac{f}{a}\right)$, $m_1m_2 = k\left(\frac{g}{b}\right)$, $n_1n_2 = k\left(\frac{h}{c}\right)$

$\therefore$ the lines are perpendicular

$\therefore l_1l_2 + m_1m_2 + n_1n_2 = 0$

$\therefore k\left(\frac{f}{a}\right) + k\left(\frac{g}{b}\right) + k\left(\frac{h}{c}\right) = 0$

i.e. $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0.$

Conclusion : Hence proved that two line al+bm+cn=0 and fmn+hlm+nlg=0 are perpendicular if $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$

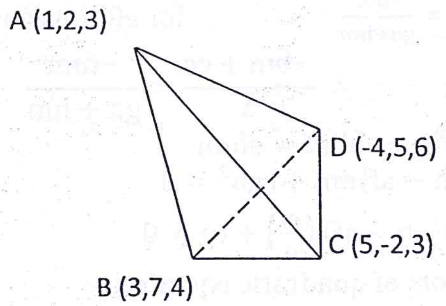
Experiment No. 8

Aim : Application of Scalar Triple Product of Vectors.

Problem 1 : Find the volume of the tetrahedron, whose vertices are $A \equiv (1,2,3)$, $B \equiv (3,7,4)$, $C \equiv (5,-2,3)$ and $D \equiv (-4,5,6)$

Formula : The volume of the tetrahedron, whose concurrent edges are A, B, AC and AD= $\frac{1}{6}$ $[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}]$

Figures :



Conclusion : Therefore the volume of the given tetrahedron is $\frac{46}{3}$ cubic units.

Experiment No. 8

Aim : Application of Scalar Triple Product of Vectors.

Problem 1 : Find the volume of the tetrahedron, whose vertices are $A \equiv (1,2,3)$, $B \equiv (3,7,4)$, $C \equiv (5,-2,3)$ and $D \equiv (-4,5,6)$

Solution : We have,

$\overrightarrow{AB} = (3-1)\hat{\imath} + (7-2)\hat{\jmath} + (4-3)\hat{k} = 2\hat{\imath} + 5\hat{\jmath} + \hat{k}$

$\overrightarrow{AC} = (5-1)\hat{\imath} + (-2-2)\hat{\jmath} + (3-3)\hat{k} = 4\hat{\imath} - 4\hat{\jmath} + 0\hat{k}$

$\overrightarrow{AD} = (-4-1)\hat{\imath} + (5-2)\hat{\jmath} + (6-3)\hat{k} = -5\hat{\jmath} + 3j + 3\hat{k}$

$$\therefore \overrightarrow{AB} \cdot \left(\overrightarrow{AC} \times \overrightarrow{AD}\right) = \begin{vmatrix} 2 & 5 & 1 \\ 4 & -4 & 0 \\ -5 & 3 & 3 \end{vmatrix} = 2(-12-0) - 5(12-0) + (12-20)$$

$$= -24 - 60 - 8$$

$$= -92$$

$\therefore$ The volume of the tetrahedron whose concurrent edges are AB, AC and AD = $\frac{1}{6}[\overrightarrow{AB}\,\overrightarrow{AC}\overrightarrow{AD}]$

$$= \frac{1}{6}(-92) = -\frac{46}{3}$$

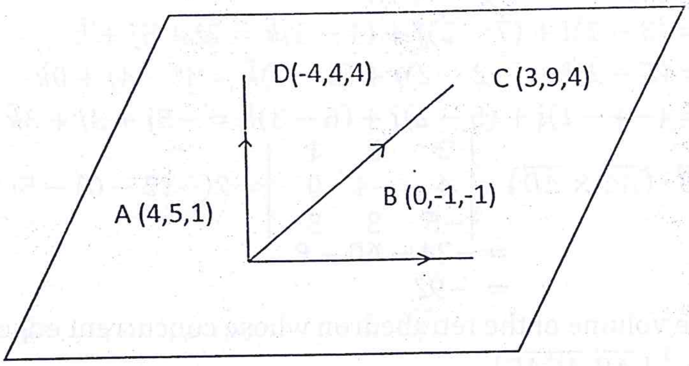
Conclusion : Therefore the volume of the given tetrahedron is $\frac{46}{3}$ cubic units.

Experiment No. 8

Problem 2 : Show that the four points $A \equiv (4,5,1), C \equiv (0,-1,-1), C \equiv (3,9,4)$ *and* $D \equiv (-4,4,4)$ are co-planar.

Formula : The four points A, B, C and D are co-planar if $[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}] = 0$

Figure :



Conclusion : Hence proved that the given four points A, B, C and D are co-planar.
A(4,5,1), B(0,-1,-1), $C \equiv (3,9,4), D \equiv (-4,4,4)$

Problem 2 : Show that the four points
$A \equiv (4,5,1),\ B \equiv (0,-1,-1),\ C \equiv (3,9,4)$ *and* $D \equiv (-4,4,4)$ are co-planar.

Solution : We have

$\overrightarrow{AB} = (0-4)\hat{\imath} + (-1-5)\hat{\jmath} + (-1-1)\hat{k} = -4\hat{\imath} - 6\hat{\jmath} - 2\hat{k}$

$\overrightarrow{Ac} = (3-4)\hat{\imath} + (9-5)\hat{\jmath} + (4-1)\hat{k} = -\hat{\imath} + 4\hat{\jmath} + 3\hat{k}$

$\overrightarrow{AD} = (-4-4)\hat{\imath} + (4-5)\hat{\jmath} + (4-1)\hat{k} = -8\hat{\imath} - \hat{\jmath} + 3\hat{k}$

$$[\overrightarrow{AB}\,\overrightarrow{AC}\,\overrightarrow{AD}] = \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = (-4)(12+3) - (-6) + (-3+24) + (-2)(1+32)$$

$$= -60 + 126 - 66$$

$\therefore\ [\overrightarrow{AB}\,\overrightarrow{AC}\,\overrightarrow{AD}] = 0$

$\therefore$ The vectors $\overrightarrow{AB}, \overrightarrow{AC}$ *and* $\overrightarrow{AD}$ are co-planar.

$\therefore$ The segments AB, AC and AD are in a plane, where A is common point of all the segments.

$\therefore$ The points A, B, C and D are co-planar.

Conclusion : Hence proved that the given four points
$A \equiv (4,5,1),\ B \equiv (0,-1,-1)\ C \equiv (3,9,4)$ *and* $D \equiv (-4,4,4)$ are co-planar.

Experiment No. 9

Aim : Three dimensional Geometry : Line

Problem 1 : Find the equations of the line passing through the point (2,0,-3) and having the direction angles 60°, 120°, 45°

Formula : $\hat{x} = a\hat{i} + b\hat{j} + c\hat{k}$

$\overrightarrow{AD} = \lambda \, \hat{r}$

$\cos 60° = \frac{1}{2}$, $\cos 120° = \frac{1}{2}$, $\cos 45° = \frac{1}{\sqrt{2}}$

Conclusion : Therefore the equations of the required line are

$x-2 = -y = \frac{z+3}{\sqrt{2}}$

Experiment No. 9

Aim : Three dimensional Geometry : Line

Problem 1 : Find the equations of the line passing through the point (2,0,-3) and having the direction angles 60°, 120°, 45°

Solution : Since the d.a.s. of the line are 60°, 120°, 45°. Therefore d.c.s. of line are $\cos 60^{\circ}, \cos 120^{\circ}, \cos 45^{\circ}$ i.e. $\frac{1}{2}, -\frac{1}{2}, \frac{1}{\sqrt{2}}$. Therefore the unit vector in direction of the line is $\hat{r} = \frac{1}{2}\hat{\imath} - \frac{1}{2}\hat{\jmath} + \frac{1}{\sqrt{2}}\hat{k}$

Let P≡ (x, y, z) be the point of line.

Since $A \equiv (2,0,-3)$ $\therefore \overrightarrow{AP} = (x-2)\hat{\imath} + y\hat{\jmath} + (2+3)\hat{k}$

i.e. $(x-2)\hat{\imath} + y\hat{\jmath} + (z+3)\hat{k} = \lambda\left(\frac{1}{2}\hat{\imath} - \frac{1}{2}\hat{\jmath} + \frac{1}{\sqrt{2}}\hat{k}\right)$

i.e. $x-2 = \frac{\lambda}{2}, y = -\frac{\lambda}{2}, z+3 = \frac{\lambda}{\sqrt{2}}$

$= x-2 = \frac{\lambda}{2}, -y = \frac{\lambda}{2}, \frac{z+3}{\sqrt{2}} = \frac{\lambda}{2}$

Conclusion : Therefore the equations of the required line are

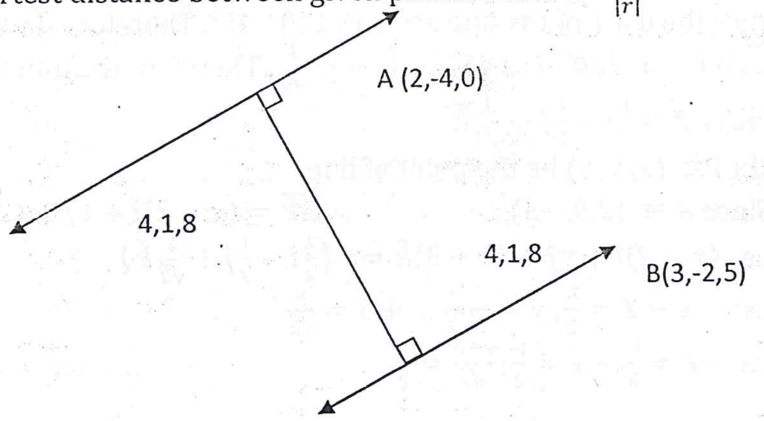
x-2= -y = $\frac{z+3}{\sqrt{2}}$

Experiment No. 9

Problem 2 : Find the shortest distance between the parallel lines $\frac{x-2}{4}=\frac{y+4}{1}=\frac{z}{8}$ and $\frac{x-3}{4}=\frac{y+2}{1}=\frac{z-5}{8}$

Formula : The shortest distance between given parallel lines is $\frac{|\overline{AB}\times\vec{R}\,|}{|r|}$

Figure :



Conclusion : The shortest distance between given parallel lines is $\sqrt{\frac{314}{9}}$ units

Experiment No. 9

Problem 2 : Find the shortest distance between the parallel line $\frac{x-z}{4}=\frac{y+4}{1}=\frac{z}{8}$ and $\frac{x-3}{4}=\frac{y+2}{1}=\frac{z-5}{8}$

Solution : The line $\frac{x-z}{4}=\frac{y+4}{1}=\frac{z}{8}$ passes through the point $A\equiv(2,-4,0)$ and the line $\frac{x-3}{4}=\frac{y+2}{1}=\frac{z-5}{8}$ passes through the point $B\equiv(3,-2,5)$ and their d.r.s. are 4,1,8.

$\therefore \overrightarrow{AB}$ = (3-2) $\hat{\imath}$ +(-2+4) $\hat{\jmath}$+(5-0) $\hat{k}$ =$\hat{\imath}$ +2$\hat{\jmath}$ + 5$\hat{k}$

And the vector along the parallel lines is $\vec{r}$ = 4$\hat{\imath}$ +$\hat{\jmath}$ + 8$\hat{k}$

$$\therefore \overrightarrow{AB}\times\vec{r}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & 2 & 5\\ 4 & 1 & 8\end{vmatrix}=(16-5)\hat{\imath}-(8-20)\hat{\jmath}+(1-8)\hat{k}$$

$$=11\hat{\imath}+12\hat{\jmath}-7\hat{k}$$

$$|\overrightarrow{AB}\times\vec{r}| = |11\hat{\imath}+12\hat{\jmath}-7\hat{k}|$$

$$=\sqrt{121+144+49}$$

$$=\sqrt{314}$$

Also $|\vec{r}| = |4\hat{\imath}+\hat{\jmath}-8\hat{k}|$

$$=\sqrt{16+1+64}$$

$$=\sqrt{81}$$

$$=9$$

$$\frac{|\overrightarrow{AB}\times\vec{r}|}{|\vec{r}|}=\frac{\sqrt{314}}{9}\text{ Units}$$

Conclusion : The shortest distance between given parallel lines is $\sqrt{\frac{314}{9}}$ units.

Experiment No. 10

Aim : Three Dimensional Geometry : Plane

Problem 1 : Find the equation of the plane passing through the point (1,2,3) and parallel to each of the lines $\frac{x-1}{1}=\frac{y-1}{0}=\frac{z-1}{3}$ *and* $\frac{x}{2}=\frac{y-1}{-3}=\frac{z}{-1}$

Formula : The vector form is $[\overrightarrow{AP}\ \vec{b}\ \vec{c}]=0$,
$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

Conclusion : The equation of plane is $9x+7y-3z-14=0$ which is passing through the point (1,2,3) and parallel
$\frac{x-1}{1}=\frac{y-1}{0}=\frac{z-1}{3}$ and $\frac{x}{2}=\frac{y-1}{3}=\frac{z}{-1}$

Experiment No. 10

Aim : Three Dimensional Geometry : Plane

Problem 1 : Find the equation of the plane passing through the point (1,2,3) and parallel to each of the lines.

$$\frac{x-1}{1} = \frac{y-1}{0} = \frac{z-1}{3} \text{ and } \frac{x}{2} = \frac{y-1}{-3} = \frac{z}{-1}$$

Solution : Consider given lines as L_1 and L_2 respectively. The d.r.s of line l_1, are 1,0,3 and of line L_2 are 2,-3,-1.

$\therefore \vec{L_1} = \hat{\imath} + 0\hat{\jmath} + 3\hat{k}$ and $L_2 = 2\hat{\imath} - 3\hat{\jmath} - \hat{k}$

Let $P \equiv (x, y, z)$ and $A \equiv (1,2,3)$ be the points of plane.

$\therefore$ d.r.s. of line AP are $x - 1, y - 2, z - 3$.

$\therefore \overrightarrow{AP} = (x-1)\hat{\imath} + (y-2)\hat{\jmath} + (z-3)\hat{k}$

Since the lines AP, L_1 and L_2 are co-planar,

$\therefore$ Vectors $\overrightarrow{AP}, \overrightarrow{L_1}$ and $\overrightarrow{L_2}$ are coplanar

$\therefore [\overrightarrow{AP}\ \overrightarrow{L_1}\ \overrightarrow{L_2}] = 0$

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & 0 & 3 \\ 2 & -3 & -1 \end{vmatrix} = 0$$

i.e. $(x-1)9 - (y-2)(-7) + (z-3)(-3) = 0$

i.e. $9x - 9 + 7y - 14 - 3z + 9 = 0$

i.e. $9x + 7y - 3z - 14 = 0$

Conclusion : The equation of line is $9x + 7y - 3z - 14 = 0$ is passing through plane the point (1,2,3) and parallel $\frac{x-1}{1} = \frac{y-1}{0} = \frac{z-1}{3}$ and $\frac{x}{2} = \frac{y-1}{3} = \frac{z}{-1}$

Experiment No. 10

Problem 2 : Find the angle between the planes $(3x - 2y + 6z - 5 = 0)$ and $2x - y - 2z - 7 = 0$

Formula : $\cos\theta = \left|\frac{\overrightarrow{n_1}\overrightarrow{n_2}}{|\overrightarrow{n_1}||\overrightarrow{n_2}|}\right|$

This is the angle between two planes.

Conclusion : The angle between two planes is $\cos^{-1}\left(\frac{4}{21}\right)$

Experiment No. 10

Problem 2 : Find the angle between the planes
$3x - 2y + 6z - 5 = 0$ and $2x - y - 2z - 7 = 0$

Solution : Let $\overrightarrow{n_1}$ *and* $\overrightarrow{n_2}$ be the normals to the planes

$3x - 2y + 6z - 5 = 0$ ---------- (1)

$2x - y - 2z - 7 = 0$ ------------------ (2) respectively

$\therefore\ \overrightarrow{n_1} = 3\hat{\imath} - 2\hat{\jmath} + 6\hat{k}$ and $\overrightarrow{n_2} = 2\hat{\imath} - \hat{\jmath} - 2\hat{k}$

Let θ be the angle between the planes (1) & (2)

$$\therefore\ \cos\theta = \left|\frac{\overrightarrow{n_1}\cdot\overrightarrow{n_2}}{|\overrightarrow{n_1}||\overrightarrow{n_2}|}\right| = \left|\frac{(3\hat{\imath}-2\hat{\jmath}+6\hat{k})\,(2\hat{\imath}-\hat{\jmath}-2\hat{k})}{\sqrt{3^2+(-2)^2+6^2}\sqrt{2^2+(-1)^2+(-2)^2}}\right|$$

$$= \left|\frac{(3)(2)+(-2)(-1)+6(-2)}{\sqrt{49}\sqrt{9}}\right|$$

$$= \left|\frac{-4}{7\times 3}\right|$$

$$= \frac{4}{21}$$

$$\therefore\ \theta = \cos^{-1}\left(\frac{4}{21}\right)$$

Conclusion : The angle between the two planes is $\cos^{-1}\left(\frac{4}{21}\right)$

Experiment No. 11

Aim : Linear Programming Problem

Example : Solve the following LPP graphically using corner point method.

Minimizing $z = 30x+20y$

Subject of constraints $x + y \leq 8$

$x + 2y \geq 4$

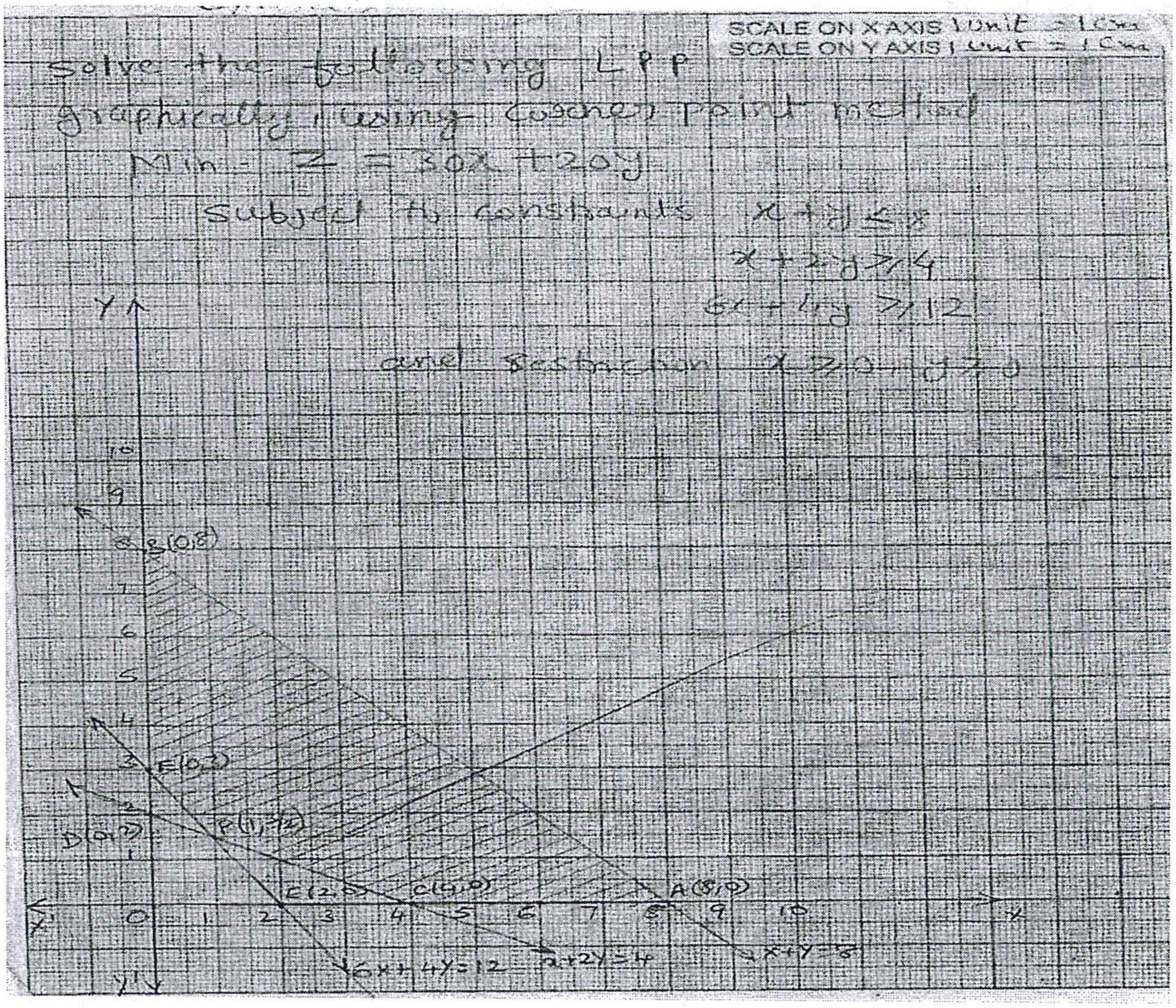
$6x + 4y \geq 12$ and

Restrictions $x \geq 0,\ y \geq 0$

Table :

Step I :

Constraints	Boundary Line	Points on Boundary line		Condition at	Side of region
		x-axis	y-axis	(0,0)	
$x + y \leq 8$	$x + y = 8$	A(8,0)	B(0,8)	$0 < 8$	Origin
$x + 2y \geq 4$	$x + 2y = 4$	C(4,0)	D(0,2)	$0 \ngtr 4$	Non-origin
$6x + 4y \geq 12$	$6x + 4y = 12$	E(2,0)	F(0,3)	$0 \ngtr 12$	Non –origin
$x \geq 0, y \geq 0$	$x = 0, y = 0$	OX-axis	OY-axis	$0 > 0, 4 > 0$	First quadrant



Conclusion : Therefore the optimum value is unique but there are infinite optimal solutions. In the case there are alternative or multiple optimal solutions.

Experiment No. 11

Aim : Linear Programming Problem

Example : Solve the following LPP graphically using corner point method.

Minimizing $z = 30x+20y$

Subject of constraints $x + y \leq 8$

$x + 2y \geq 4$

$6x + 4y \geq 12$ and

Restrictions $x \geq 0,\ y \geq 0$

Solution :

Step 1 : The table of information about the constraints for drawing a feasible region is as follows :

Step 2 : Using the above information the feasible region is as follows : The shaded region in ALPEB is the feasible region. In this case the line of objective function is parallel to the feasible region formed by one of the constraints.

Step 3 : The point of intersection of boundary lines CD and EF is given by

6x-2x=12-8 i.e. 4x=4 $\therefore x = 1$

And $y = \frac{3}{2}$ $\therefore FPCABF$ is feasible region

The co-ordinates of vertices of feasible region are $A \equiv (8,0), C \equiv (4,0),$

$P \equiv \left(1, \frac{3}{2}\right),$ $F \equiv (0,3), B \equiv (0,8)$

Step 4 : The value of objective function $Z = 30x + 20y$

At the vertex A is $z(8,0) = 30(8) + 20(0) = 240$

At the vertex C is $z(4,0) = 30(4) + 20(0) = 120$

At the vertex P is $z\left(1, \frac{3}{2}\right) = 30(1) + 20\left(\frac{3}{2}\right) = 60$

At the vertex F is $z(0,3) = 30(0) + 20(3) = 60$

At the vertex B is $z(0,8) = 30(0) + 20(8) = 160$

Step 5 : The value of objective function Z is minimum 60 at two vertices P and F of the feasible region. Thus at all the points of the segment P & F of the boundary line, the minimum value of Z is same i.e. 60.

Conclusion : Therefore the optimum value is unique but there are infinite optimal solutions. In the case there are alternative or multiple optimal solutions.

Experiment No. 11

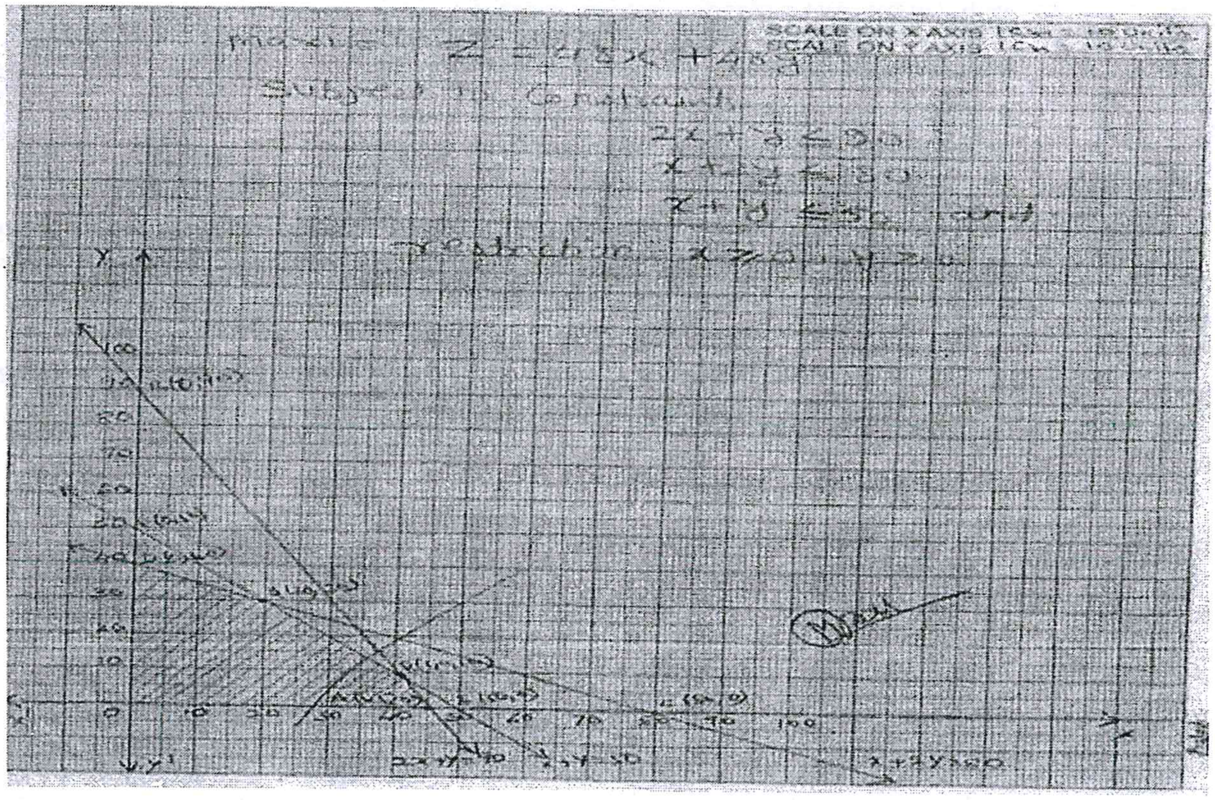
Problem 2 : A carpenter has 90, 80 and 50 running feet of teak, plywood and rosewood respectively. the product A requires 1, 2 and 1 running feet of teak, plywood and rose wood respectively. If A would be sold for Rs. 48 per unit and B would be sold for 40 per unit, how much of each should be make and sold in order to obtain the maximum income out of his stock of wood? Formulate the mathematical method for this linear programming problem and hence solve it graphically, using corner point method.

Table II : Step I :

Type of wood	Requirement		Decision variable	Relation	Maximum availability
	Product A	Product B			
Teak	2	1	x	≥	90
Plywood	1	2	y	≥	80
Rosewood	1	1		≥	50
Income	[48	40]		=	z

Step 3 :

Constraints	Boundary Line	Points on Boundary line		Condition at	Side of region
		x-axis	y-axis	(0,0)	
$2x + y \leq 90$	$2x + y = 90$	A(45,0)	B(0,90)	$0 < 90$	Origin side
$x + 2y \leq 80$	$x + 2y = 80$	C(80,0)	D(0,40)	$0 < 80$	Origin side
$x + y \leq 50$	$x + y = 50$	E(50,0)	F(0,50)	$0 < 50$	Origin side
$x \geq 0, y \geq 0$	$x = 0, y = 0$	0 X-axis	Y-axis	$0 > 0, y > 0$	First quadrant



Conclusion : Hence the maximum income is Rs. 2,320 when he produces 20 units of product A and 30 units of product B.

Problem 2 : A carpenter has 90, 80 and 50 running feet of teak, plywood and rosewood respectively. the product A requires 2, 1 and 1 running feet of teak, plywood and rose wood respectively. If A would be sold for Rs. 48 per unit and B would be sold for 40 per unit, how much of each should be make and sold in order to obtain the maximum income out of his stock of wood? Formulate the mathematical method for this linear programming problem and hence solve it graphically, using corner point method.

Solution :

Step 1 : Let the carpenter produces x units of product A and Y units of product B

$\therefore x \geq 0, y \geq 0$

The information of requirements and availability of raw material for the production of A and B products is given in following.

← tabular form.

Step 2 : For making x unit of product A and Y units of product B.

a) $2x + y$ but teak is required and 90 feet teak is available

$\therefore 2x + y \leq 90$

b) $x + 2y$ feet plywood is required and 80 feet plywood is available

$\therefore x + 2y \leq 80$

c) $x + y$ feet rosewood is required and 50 feet rosewood is available

$x + y \leq 50$

Since the selling price of one unit of product A is Rs. 48 and of product B is Rs. 40

Therefore total selling price of x units of product A and Y units of product B is 48x+ 40y. It is to be maximized.

Therefore the mathematical formulation of L.P.P. problem is maximized

$Z = 48x + 40y$

Subject to constraints $2x + y \leq 90$

$x + 2y \leq 80$

$x + y \leq 50$

And restriction $x \geq 0, \ y \geq 0$

Step 3 : Now collect the information required for drawing the feasible region as follows :

Step 4 : Using the above information, we have the feasible region as OARQD. Now solving the equation $2x + y = 90$ of line AB with equation $x + y = 50$ of line ET, we get the co-ordinates of point of intersection R as (40,10)

Also solving the equation $x + 2y = 80$ of line CD with the equation $x + y = 50$ of line EF, we get the co-ordinates of their point of intersection as Q (20, 30)

Therefore points $O \equiv (0,0),\ A \equiv (45,0),\ R \equiv (40,10)$

$Q \equiv (20,30)$ *and* $D \equiv (40,0)$ are corner points of feasible region.

Step 5 : The value of the objective function. $z = 48x + 40y$

at O is $z(0,0) \equiv 48(0) + 40(0) = 0$

at A is $z(45,0) \equiv 48(45) + 40(0) = 2160$

at R is $z(40,10) \equiv 48(40) + 40(10) = 2320$

at Q is $z(20,30) \equiv 48(20) + 40(30) = 2160$

at D is $z(48,40) \equiv 48(0) + 40(40) = 1600$

Therefore z is maximum at $R \equiv (40, 10)$

Conclusion : Hence the maximum income is Rs. 2,320 when he produces 20 units of product A and 30 units of product B.

Experiment No. 12

Aim : Applications of Derivatives- Tangent and Normal.

Problem 1 : Find the equation of tangent and normal to circle $x^2 + y^2 - 3x + 4y - 31 = 0$ at the point $(-2,3)$

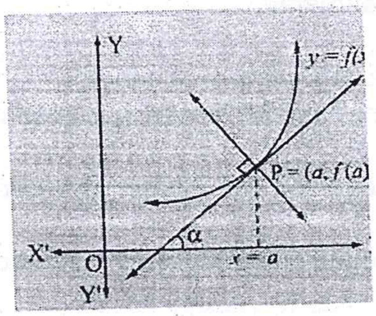
Formula : $y = u \pm v$

Then $\frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$

Equation of tangent is $y - f(a) = f'(a)(x - a)$

Equation of normal is $y - f(a) = \frac{-1}{f'(a)}(x - a)$

Figure:



Conclusion : The equation of tangent and normal are $7x - 10y + 44 = 0$ and $10x + 7y - 1 = 0$

Experiment No. 12

Aim : Applications of Derivatives- Tangent and Normal.

Problem 1 : Find the equation of tangent and normal to circle $x^2 + y^2 - 3x + 4y - 31 = 0$ at the point $(-2,3)$

Solution : The given equation of circle

$x^2 + y^2 - 3x + 4y - 31 = 0$ -------------- (1)

Now differentiating equation (1) w.r.t. x, we get

$2x + 2y\frac{dy}{dx} - 3 + 4\frac{dy}{dx} = 0$

$\therefore 2\frac{dy}{dx}(y + 2) = -(2x - 3)$ $\qquad \because \frac{dy}{dx} = \frac{-(2x-3)}{2(y+2)}$

$\therefore \left|\frac{dy}{dx}\right|_{(-2,3)} = \frac{-\{2(-2)-3\}}{2(y+2)} = \frac{7}{10}$

$\therefore$ Gradient of curve (1) at (-2,3)$= \frac{7}{10} =$ slope of tangent to the curve (1) at (-2,3)=7/10

Therefore equation of tangent to circle (1) at (-2, 3) is $y - 3 = \frac{7}{10}(x + 2)$

i.e. $10y - 30 = 7x + 14$ $\quad i.e.\ 7x - 10y + 44 = 0$

the slope of normal$= \frac{-1}{slope\ of\ tangent} = \frac{-10}{7}$

Therefore equation of normal to circle (1) at (-2,3) is

$y - 3 = \frac{-10}{7}(x + 2)$ $\quad i.e.\ 7y - 21 = -10x - 20$

i.e. $10x + 7y - 1 = 0$

Conclusion : The equation of tangent and normal are

$7x - 10y + 44 = 0$ $\quad and\quad 10x + 7y - 1 = 0$

Experiment No. 12

Problem 2 : Find the points on the curve $y = x^3 - 9x^2 + 15x + 3$ at which tangents are parallel to x-axis.

Formula :

If y= $y = u \pm v$

Then $\frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$

Conclusion : The points on curve at which tangents are parallel to x-axis are y-curves $(1,10)$ *and* $(5,-22)$

Experiment No. 12

Problem 2 : Find the points on the curve $y = x^3 - 9x^2 + 15x + 3$ at which the tangents are parallel to x-axis.

Solution : The given curve is $y = x^3 - 9x^2 + 15x + 3$ --------- (1)

Now differentiating curve (1), we get,

$\frac{dy}{dx} = 3x^2 - 18x + 15$ $\quad \therefore \frac{dy}{dx} = 3(x^2 - 6x + 5)$

$\therefore \frac{dy}{dx} = 3(x-5)(x-1)$

Since the tangent is parallel to x-axis.

Therefore the slope of tangent $= \frac{dy}{dx} = 0$

$\therefore (x-5)(x-1) = 0 \quad \therefore x = 1 \ \ or\ x = 5$

From (1), for $x = 1,\ y = 1 - 9 + 15 + 3$ i.e. y=10 and

For $x = 5,\ y = 125 - 225 + 75 + 3 = -22$

Hence, the required points on i.e. y curves are (1,10) and (5,-22).

Conclusion : The points on curve at which tangents are parallel to x-axis are y-curves $(1,10)$ *and* $(5, -22)$

Experiment No. 13

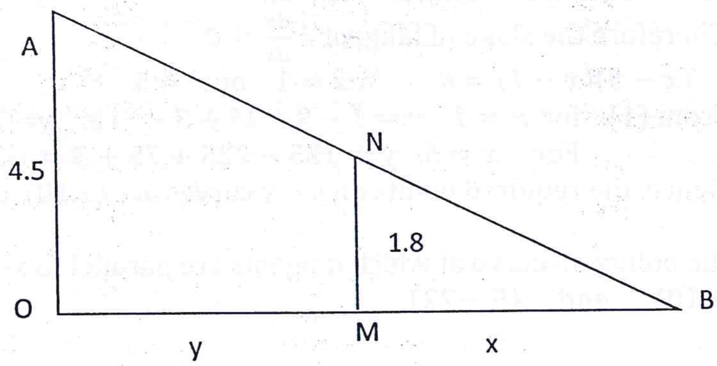
Aim : Applications of Derivatives- Rate Measure

Problem 1 : A man of height 180 cm is moving away from a lamp post at a rate of 1.2 m/s. If the height of the lamp post is 4.5 m, find the rate at which (i) his shadow is lengthening (ii) the tip of his shadow is moving.

Formula : if $x + y$,

Then $\frac{d}{dt}(x + y) = \frac{dx}{dt} + \frac{dy}{dt}$

Figure :



Conclusion : The shadow is lengthening at the rate of 0.8 m/sec and tip of the shadow is moving away from the lamp post at the rate of 2 m/s.

Experiment No. 13

Aim : Applications of Derivatives- Rate Measure

Problem 1 : A man of height 180 cm is moving away from a lamp post at a rate of 1.2 m/s. If the height of the lamp post is 4.5 m, find the rate at which (i) his shadow is lengthening (ii) the tip of his shadow is moving.

Solution : Let OA = 4.5m be the height of lamp post, MN= 1.8 m be the height of the man.
Let the man be y meters away from lamp post and x meters be the length of his shadow at time t.

$\therefore\ OM = y \quad and\ MB = x$

$\therefore$ the rate at which the man is moving away from lamp post is $\frac{dy}{dt} = 1.2$

The rate at which his shadow is lengthening is $\frac{dx}{dt}$

$\therefore$ The rate at which the tip of the shadow is moving $= \frac{d}{dt}(x+y)$

$= \frac{dx}{dt} + \frac{dy}{dt}$

From the figure $\Delta\, OAB$ and ΔMNB are similar

$\therefore \frac{x}{1.8} = \frac{x+y}{4.5} \qquad \therefore 45x = 18x + 18y \qquad \therefore x = \frac{2}{3}y$

$\frac{dx}{dt} = \frac{2}{3}\frac{dy}{dt} = \frac{2}{3}(1.2) = 0.8$ and

$\frac{dx}{dt} + \frac{dy}{dt} = 0.8 + 1.2 = 2$

Conclusion : The shadow is lengthening at the rate of 0.8 m/sec and tip of the shadow is moving away from the lamp post at the rate of 2 m/s.

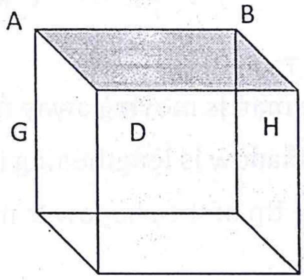
Experiment No. 13

Problem 2 : A metal cube expands under heating 80 that its increase by 2%. Find the approximate increase in the volume of metal cube, if its side before heating is 10 cm.

Formula : Volume of cube = a^3

Percentage = $\frac{\delta a}{a} \times 100$

Figure :



Conclusion : Approximate increase in the volume is $60cm^3$

Experiment No. 13

Problem 2 : A metal cube expands under heating so that its side increases by 2%. Find the approximate increase in the volume of the metal cube, its side before heating is 10cm.

Solution : Let V be the volume of the metal cube of side x cm.

$\therefore v = x^3 \qquad \therefore \frac{dv}{dx} = 3x^2$ and

$\left[\frac{dv}{dx}\right]_{x=10} = 3(10)^2 = 300$

Let δx be the increase in side length.

$\delta x = 2\% \ \ of\ x = \frac{2}{100}(x) = \frac{2}{100}(10)$

$\therefore \delta x = 0.2\ cm$

Let δv be the increase in volume

$\therefore \ \delta v = \frac{dv}{dx}\delta x \qquad \therefore \delta v = (300)(0.2) = 60$

Conclusion : Approximate increase in the volume is 60 cm^3.

Experiment No. 14

Aim : Applications of Derivatives- Maxima and Minima.

Problem 1 : An open tank is to be constructed with a square base and vertical sides as to contain 500 cube meters of water. What should be the dimension of the tank if the area of metal sheet used in its construction is to be minimum.

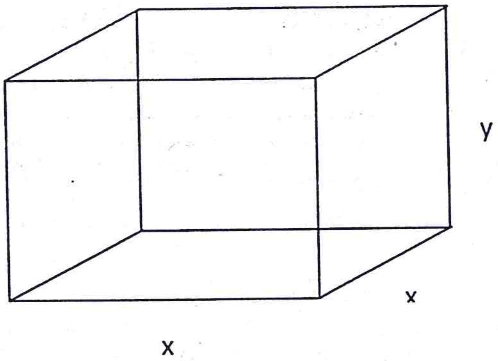
Formula : Volume of tank = x^2y

x= length y= height

surface area of open tank

$$A = (area\ of\ base) \times 4(area\ of\ a\ side)$$

Figure :



Conclusion : The side of square base is of length 10 mts and height is of 5 mts. So that the area of metal used is minimum

Experiment No. 14

Aim : Applications of Derivatives- Maxima and Minima.

Procedure : An open tank is to be constructed with a square base and vertical sides as to contain 500 cube meters of water. What should be the dimension of the tank if the area of metal sheet used in its construction is to be minimum.

Solution : Let x mts be the length of the side of the square base and y mts be the height of the open tank.

$\therefore$ volume of the tank = x^2y= 500

$\therefore y = \frac{500}{x^2}$

Let A be the surface area of open tank

$\therefore A = (area\ of\ base\ ) + 4\ (area\ of\ a\ side)$

$A = x^2 + 4xy = x^2 + 4x\left(\frac{500}{x^2}\right)$ $\qquad$ $A = x^2 + \frac{2000}{x}$

$\frac{dA}{dx} = 2x - \frac{2000}{x^2}$ $\qquad$ $\frac{dA}{dx} = 2\left(\frac{x^3-1000}{x^2}\right)$

Let $\frac{dA}{dx} = 0$ $\qquad$ $\therefore \frac{x^3-1000}{x^2} = 0$

i.e. $x^3 = 1000$

$\therefore x = 10$

$\frac{d^2A}{dx^2} = 2\left(1 + \frac{2000}{x^3}\right)$

Now $\left.\frac{d^2A}{dx^2}\right|_{x=10} = 2\left(1 + \frac{2000}{x^3}\right) = 6 > 0$

$\therefore A$ is minimum, when x= 10 mts.

Also, when x=10, y=$\frac{500}{x^2} = \frac{500}{100} = 5$

Conclusion : The side of square base is of length 10 mts and heights is of 5 mts. So that the area of metal used is minimum.

Experiment No. 14

Problem 2 : Twenty meters of wire is available to fence off a flower bed in the form of a circular sector. What must be the radius of the circle if we wish to have flower bed with the greatest possible surface area?

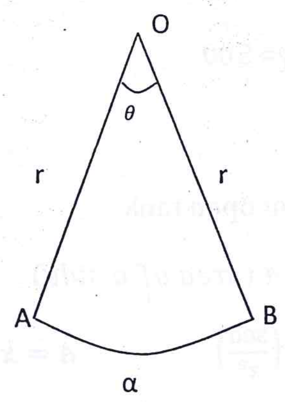
Formula : For a circular sector,

Length of arc $A \times B = r\theta,$

Perimeter of sector$= r + r + r\theta$

Area of sector $f(r) = \frac{1}{2}r^2\theta$

Figure :



Conclusion : For greatest possible surface area of flower bed, its radius must be 5 cm.

Experiment No. 14

Problem 2 : Twenty meters of wire is available to fence off a flower bed in the form of a circular sector. What must be the radius of the circle if we wish to have a flower bed with the greatest possible surface area?

Solution : Let r be radius of circular sector

$\therefore$ length of arc $A \times B = r\theta$

$\therefore$ perimeter of sector $= r + r + r\theta$

$\therefore 20 = 2r + r\theta$ $\qquad$ $\therefore (20 - 2r) = r\theta$ $\qquad$ $\theta = \frac{20-2r}{r}$

Now, $\qquad$ Area of sector $f(r) = \frac{1}{2} r^2\theta$

$\therefore f(r) = \frac{1}{2}r^2\left(\frac{20-2r}{r}\right)$ $\qquad$ $\therefore f(r) = \frac{1}{2}r(20 - 2r) = 10r - r^2$

$\therefore f'(r) = 10 - 2r = 2(5 - r)$

$\therefore f'(r) = 0$ $\qquad$ $\therefore 2(5 - r) = 0$

$\therefore f''(r) = -2$ $\qquad$ $\therefore f''(5) = -2 < 0$

$\therefore$f(r) has maximum at r=5

For greatest possible surface area of flower bed, its radius must be 5 cm.

Conclusion : For greatest possible surface area of flower bed, its radius must be 5 cm.

Experiment No. 15

Aim : Applications of Derivatives- Rolle's Theorem and LMVT

Problem 1 : If the function $f(x) = (x-2)\ \log x$, then show that $x\ \log x = 2 - x$ has a root between 1 & 2.

Formula : Rolle's Theorem : If a function $f(x)$ is

i. Continuous in the closed interval [a,b]
ii. Differentiable in the open interval [a,b] and
iii. $f(a) = f(b)$, then there exists at least one value c of x in the open interval [a,b] such that $f'(c) = 0$

Conclusion : Therefore, the equation $x \log x = 2 - x$ has a root in (1,2)

Experiment No. 15

Aim : Applications of Derivatives Rolle's Theorem and LMVT.

Problem 1 : If the function $f(x) = (x-2)\ \log x$, then show that $x \log x = 2 - x$ has a root between 1 & 2.

Solution : The given function is $f(x) = (x-2)\log x$

i. Since the polynomial function x -2 is continuous on R and log x is continuous on $(0, \infty)$ therefore the product function $f(x) = (x-2)\log x$ is continuous on common interval [1,2]

ii. Now $f'(x) = (x-2)\left(\frac{1}{x}\right) + \log x\,(1) = 1 - \frac{2}{x} + \log x$ and it has unique value in [1,2]
$\therefore f(x)$ is differentiable in [1,2]

iii. $f(1) = (1-2)\log(1) = (-1)(0) = 0,$

$f(2) = (2-2)\log 2 = 0$

$\therefore f(1) = f(2)$
$\therefore f(x)$ satisfies all the conditions of Rolle's theorem.
$\therefore$ There is a value C in (1,2) such that $\therefore f'(c) = 0$
$\therefore 1 - \frac{2}{c} + \log c = 0$ $\quad\quad$ $\therefore$ $c \log c = 2 - c$ and c is in (1,2)
$\therefore$ The equation $\therefore x\ \log x = 2 - x$ has a root in (1,2)

Conclusion : Therefore, the equation $x \log x = 2 - x$ has a root in (1,2)

Experiment No. 15

Problem 2 : Verify LMVT for function $f(x) = (x-2)(x-3)(x-5)$ on $[0,5]$

Formula : Lagrange's Mean Value Theorem (LMVT). If a function f(x) is

i) Continues in closed interval [a,b],
ii) Differentiable in the open interval (a,b), then there exists at least value of x in the open interval (a,b) such that $\frac{f(b)-f(a)}{b-a} = f'(c)$

Conclusion: Since $\frac{5}{3} \in (0,5) \therefore$ for function $f(x) = (x-2)(x-3)(x-5)$

LMVT is verified.

Experiment No. 15

Problem 2: Verify LMVT for the function $f(x) = (x-2)(x-3)(x-5)$ on $[0,5]$.

Solution : Since $f(x)= (x-2)(x-3)(x-5)$

$\therefore f(x) = x^3-10x^2+31x-30$

(i) Since f(x) is a polynomial function. Therefore it is continuous on R. Thus f(x) is continuous on [0,5].

(ii) $f'(x)=3x^2-20x+31$ and it has unique value at any point in interval on [0,5]

$\therefore$ f(x) is differentiable on [0,5]

Thus f(x) satisfies the conditions of LMVT on [0,5]

$\therefore$ There exists one value $c \in (0,5)$ such that $\frac{f(5)-f(0)}{5-0} = f'(c)$

Now $f(a) = f(0) = (0-2)(0-3)(0-5)= -30$

$f(b) = f(5) = (5-2)(5-3)(5-5)= 0$

and $f'(c) = 3c^2 -20c +31$

$\therefore \frac{0-(-30)}{5-0} = 3c^2 - 20x +31$ $\qquad \therefore 3c^2-20x+25 = 0$

$\therefore c = \frac{+20\pm\sqrt{(-20)^2-4(3)(25)}}{2(3)} = \frac{+20\pm 10}{6} = 5, \frac{5}{3}$

Since $\frac{5}{3} \in (0,5)$ $\qquad \therefore$ LMVT is verified.

Conclusion : Since $\frac{5}{3} \in (0,5)$ $\therefore$ for function $f(x) = (x-2)(x-3)(x-5)$ on $[0,5]$. LMVT is verified.

Experiment No. 16

Aim : Applications of Definite Integrals as Limit of Sum.

Problem 1 : Express $\int_0^1 (3x^2+2)\,dx$ as a limit of a sum and evaluate it. Also compare the answer by actually evaluating it.

Formula :

i. $\int_a^b f(x)dx = (b-a)\, lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} f(a+rh)$

$\int_a^b f(x)dx = lim_{h\to 0}\, h\frac{1}{n}\sum_{r=1}^{n} f(a+rh)$, where $\frac{b-a}{h} = h$

ii. Let $[0,1] \equiv [a,b]$, $a=0$ and $b=1$

$\therefore \frac{1-0}{h} = h \quad \therefore h = \frac{1}{n}$ and $nh=1$

Also $a + rh = 0 + rh = rh = \frac{r}{h}$

$\int_0^1 f(x)dx = \lim_{h\to 0} \sum_{r=1}^{n} f(rh) = \lim_{h\to\infty} \sum_{a=1}^{n} f\left(\frac{r}{h}\right),$

iii. $\sum_{r=1}^{n} r^2 = \frac{1}{6} n(n+1)(2n+1)$ where nh=1

Conclusion : we observe that the value of the integral in both the method is same .

$\int_0^1 (3x^2+2)\,dx = 3$

Experiment No. 16

Aim : Applications of Define Integrals as Limit of Sum.

Problem 1 : Express $\int_0^1 (3x^2 + 2)\, dx$ as a limit of a sum and evaluate it. Also compare the answer by actually evaluating it.

Solution : Let $a = 0, b = 1$ *and* $f(x) = 3x^2 + 2$

$\therefore \quad nh = 1 \quad and \quad f(rh) = 3(nh^2) + 2$

Since $\int_0^1 f(x)\, dx = \lim_{h\to 0} h \sum_{r=1}^{n} f(rh)$

$\therefore \int_0^1 (3x^2 + 2)\, dx = \lim_{h\to 0} h \sum_{r=1}^{h} [3(rh)^2 + 2]$

$= \lim_{h\to 0} h \{3h^2 \sum_{r=1}^{n} r^2 + 2 \sum_{r=1}^{n} 1\}$

$= \lim_{h\to 0} h \left\{\frac{3h^2(n(n+1)(2n+1))}{6} + 2n\right\}$

$= \lim_{h\to 0} \left\{\frac{1}{2}(nh)(nh + h)(2nh + h) + 2nh\right\}$

$= \lim_{h\to 0} \left\{\frac{1}{2}(1)(1 + h)(2 + h) + 2\right\} = \frac{1}{2}(1)(1)(2) + 2 = 3$

--------------- (1)

$\int_0^1 (3x^2 + 2)dx = [x^3 + 2x]_0^1 = [(1)^3 + 2(1)] - [(0)^3 + 2(0)] = 3$

From (1) & (2), we observe that value of integral in both the methods is same.

Conclusion : we observe that value of integral in both the methods is same.

$\int_0^1 (3x^2 + 2)dx = 3$

Experiment No. 16

Problem 2: Express $\int_0^{\frac{\pi}{2}} \sin x\, dx$ as a limit of sum and evaluate it.

Formula :

i. $\int_a^b f(x)dx = \lim_{h\to 0} h \sum_{r=1}^{n} f(rh)$

$= \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} f\left(\frac{r}{n}\right)$

ii. $\int_a^b f(x)dx = \lim_{h\to 0} h \sum_{r=1}^{n} f(a+rh)$

iii. $2\sin(r)\sin\left(\frac{h}{2}\right) = \cos\frac{h}{2}(2r-1) - \cos\frac{h}{2}(2r+1)$

iv. $\lim_{h\to 0} \frac{h/2}{\sin(h/2)} = 1$

Conclusion : Therefore the solution of $\int_0^{\pi/2} \sin x\, dx = 1$.

Experiment No. 16

Problem 2 : Express $\int_0^{\pi/2} \sin x\, dx$ as a limit of a sum and evaluate it.

Solution : Let $\int_0^{\pi/2} \sin x\, dx = \int_a^b f(x)dx$

$\therefore a = 0, b = \frac{\pi}{2}$ and $f(x) = \sin x$

$\therefore n = \frac{b-a}{h} = \frac{\pi/2 - 0}{h} \qquad nh = \frac{\pi}{2}$

Also $\quad f(a+rh) = f(rh) = \sin(rh)$

Since $\quad \int_a^b f(x)dx = \lim_{h\to 0} \sum_{r=1}^{n} f(a+rh)$

$$\therefore \int_0^{\frac{\pi}{2}} \sin x\, dx = \lim_{h\to 0} \frac{h}{2\sin\left(\frac{h}{2}\right)} \sum_{r=1}^{n} 2\sin(rh)\sin\left(\frac{h}{2}\right)$$

$$= \lim_{h\to 0} \frac{h}{2\sin\left(\frac{h}{2}\right)} \sum_{r=1}^{n} \left\{\cos\frac{h}{2}(2r-1) - \cos\frac{h}{2}(2r+1)\right\}$$

$$= \lim_{h\to 0} \frac{h}{2\sin\left(\frac{h}{2}\right)} \left\{\left|\cos\left(\frac{h}{2}\right) - \cos\left(\frac{3h}{2}\right)\right| \left|\cos\left(\frac{3h}{2}\right) - \cos\left(\frac{3h}{2}\right)\right| + \right.$$

$$\left. \ldots + \left|\cos\frac{h}{2}(2n-1) - \cos\frac{h}{2}(2n+1)\right|\right\}$$

$$= \lim_{h\to 0} \frac{h/2}{\sin\left(\frac{h}{2}\right)} \left\{\cos\left(\frac{h}{2}\right) - \cos\frac{h}{2}(2n+1)\right\}$$

$$= \lim_{h\to 0} \frac{h/2}{\sin(h/2)} \left\{\cos\left(\frac{h}{2}\right) - \cos\left(\frac{\pi}{2} + \frac{h}{2}\right)\right\}$$

$$\int_0^{\frac{\pi}{2}} \sin x\, dx = (1)\left(\cos 0 - \cos\frac{\pi}{2}\right)$$

$$\int_0^{\frac{\pi}{2}} \sin x\, dx = 1$$

Conclusion : Therefore the solution of $\int_0^{\pi/2} \sin x\, dx$ is 1.

Experiment No. 17

Aim : Applications of Definite Integrals : Area

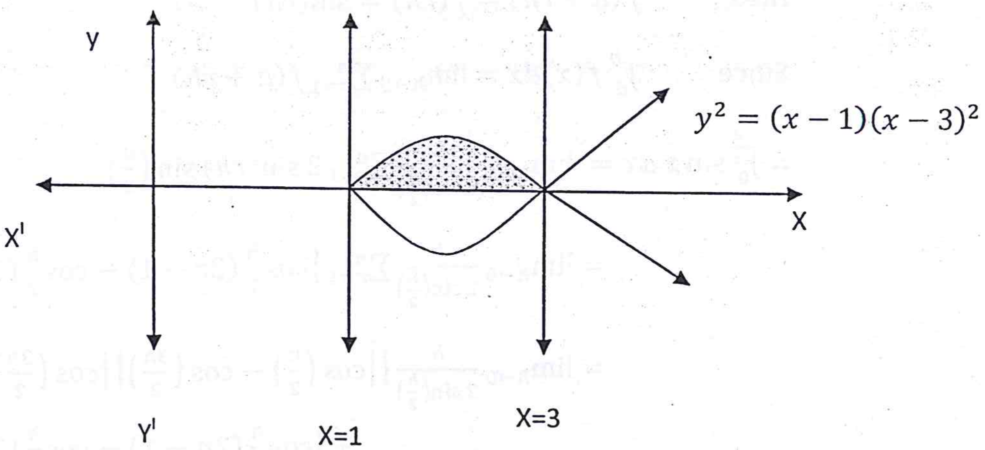
Problem 1: Find the area of the loop of the curve $y^2 = (x-1)(x-3)^2$.

Formula :

i. Area of loop = 2 [area bounded by curve y]
$= (x-3)(\sqrt{x-1})$, X- axis and the co-ordinates x=1 and x=3)

ii. $\int_a^b [f_1(x) \pm f_2(x)]dx = \int_a^b f_1(x)dx \pm \int_a^b f_2(x)dx$

Figure :



Conclusion : [Sign of area is always positive]

$\therefore$ Area of loops $= \frac{32\sqrt{2}}{15}$ sq. units

Experiment No. 17

Aim : Applications of Definite Integrals.

Problem 1 : Find the area of the loop of the curve $y^2 = (x-1)(x-3)^2$

Solution : Since for points $x - y$ and (x, y) the equation $y^2 = (x-1)(x-3)^2$ of curve does not change. Therefore the curve is symmetric about x-axis, for $y = 0, (x-1)(x-3)^2 = 0$

$\therefore x = 1 \; or \; 3$

Also, $y = (x-3)(\sqrt{x-1})$

Draw the loop of the curve

Area of the loop= 2 [area bounded by the curve y]

$=(x-3)(\sqrt{x-1})$, x-axis and the co-ordinates

$x = 1$ and $x = 3$]

$=2\int_1^3 (x-3)\sqrt{x-1}dx$

Let $x - 1 = t^2$ $\therefore dx = 2t\,dt$

For $x = 1, t = 0$ and for $x = 3, t = \sqrt{2}$

Since $x = 1 + t^2$ $\therefore (x-3)\sqrt{x-1} = (1 + t^2 - 3)t = t^3 - 2t$

Area of loop $= 2\int_0^{\sqrt{2}}(t^3 - 2t)2t\,dt = 4\int_0^{\sqrt{2}}(t^4 - 2t^2)dt = 4\left|\frac{t^5}{5} - \frac{2t^3}{3}\right|_0^{\sqrt{2}}$

$$= 4\left|\frac{(\sqrt{2})^5}{5} - \frac{2(\sqrt{2})^3}{3} - 0 + 0\right| = 4\left|\frac{4\sqrt{2}}{5} - \frac{4\sqrt{2}}{3}\right| = \frac{32\sqrt{2}}{15}$$

Conclusion : Area of loops $= \frac{32\sqrt{2}}{15}$ sq. units

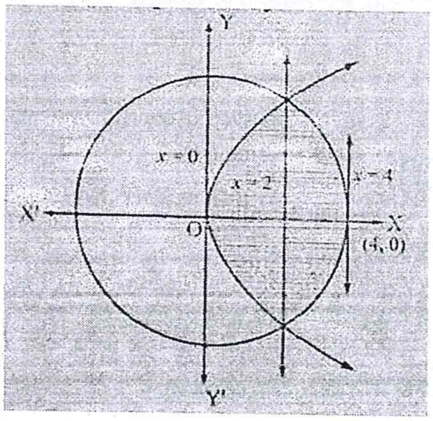
Experiment No. 17

Problem 2 : Find the area bounded by the circle $x^2 + y^2 = 16$ and the parabola $y^2 = 6x$

Formula : Area bounded by circle (1) and parabola (2)

$$= \int \sqrt{a^2 - x^2} = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{4}\right)$$

Figure :



$x^2 + y^2 = 16$

$y^2 = 6x$

Conclusion : The area bounded by the circle and the parabola is $\frac{4}{3}(4\pi + \sqrt{3})$ sq. units.

Experiment No. 17

Problem 2 : Find the area bounded by the circle $x^2 + y^2 = 16$ and of parabola is $y^2 = 6x$

Solution: The given equation of a circle is $x^2 + y^2 = 16$ -------------- (1)

and of parabola is $y^2 = 6x$ ------------ (2)

Now solving equations (1) & (2), we have

$x^2 + 6x = 16$ i.e. $x^2 + 6x - 16 = 0$

$(x + 8)(x - 2) = 0$ $\therefore x = -8$ *or* 2

Since $x \neq -8$ $\therefore x = 2$

$\therefore$ Curves (1) and (2) intersect at $x = 2$

The circle $x^2 + y^2 = 16$ is having the center at origin (0,0) and radius 4 units and it is symmetric about both the axes. The parabola is symmetric about x-axis, its vertex is at the origin (0,0) and passes through the points $\left(\frac{3}{2}, 3\right)$ *and* $\left(\frac{3}{2}, -3\right)$

$\therefore$ The area bounded by circle (1) and parabola (2)=2 [area bounded by circle (1) and parabola (2)above x-axis]

The area bounded by circle (1) and parabola (2) above x-axis is same as below x-axis.

$\therefore$ Area bounded by circle (1) and parabola (2)

$$= 2\left|\int_0^2 \sqrt{6x}\, dx + \int_2^4 \sqrt{16 - x^2}\, dx\right| = 2\left|\sqrt{6}\int_0^2 (x^{1/2})dx + \int_2^4 \sqrt{4^2 - x^2}dx\right|$$

$$= 2\left\{ \left|\sqrt{6}x^{3/2} \times \frac{2}{3}\right|_0^2 + \left|\frac{x}{2}\sqrt{4^2 - x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right|_2^4 \right\}$$

$$= 2\left\{ \sqrt{6} \times \frac{2}{3} \left|(2)^{\frac{3}{2}} - (0)^{\frac{3}{2}}\right| - \left|\left(\frac{4}{2}\sqrt{4^2 - 4^2} + \frac{16}{2}\sin^{-1}\frac{4}{4}\right) - \left(\frac{2}{2}\sqrt{4^2 - 2^2} + \frac{16}{2}\sin^{-1}\left(\frac{2}{4}\right)\right)\right| \right\}$$

$$= 2\left\{\left|\sqrt{3} \times \frac{2}{3} \times 4 - 0\right| \left|0 + 8 \times \frac{\pi}{2} - 2\sqrt{3} - 8 \times \frac{\pi}{6}\right|\right\}$$

$$= 2\left(\frac{8}{3}\sqrt{3} - 2\sqrt{3} + 4\pi - \frac{4\pi}{3}\right)$$

$$= 2\left(\frac{2}{3}\sqrt{3} + 8\frac{\pi}{3}\right)$$

$$= \frac{4}{3}(4\pi + \sqrt{3}) \text{ sq. units.}$$

Conclusion : The area bounded by the circle and the parabola is $\frac{4}{3}\left(4\pi + \sqrt{3}\right)$ sq. units.

Experiment No. 18

Aim : Applications of Differential Equations.

Problem 1 : The population of a town increases at the rate proportional to the population existing at that time. The population of the town was 5,00,000 in year 1980 and 8,00,000 in year 1990 what will be population of town in year 2010?

Tabular Form :

Time (years)t	T_0= 1980	T_1=1990	T=2010
Population	X_0=5,00,000	X_1=8,00,000	X=?

Formula : $\frac{dx}{dt} \propto x$ i.e. $\frac{dx}{dt} = kx$

$$\frac{\log x - \log x_0}{\log x - \log x_0} = \frac{t - t_0}{t_1 - t_0}$$

$$k = \left(\frac{\log_{10} x_1 - \log_{10} x_0}{t_1 - t_0}\right)$$

Conclusion : The population of the town in year 2010 will be 20,48,000.

Experiment No. 18

RHS

Aim : Applications of Differential Equations.

Problem 1 : The population of a town increases at the rate proportional to the population existing at that time. The population of the town was 5,00,000 in year 1980 and 8,00,000 in year 1990 what will be population of town in year 2010?

Solution : Let x be population of town at time t years.

$\therefore \frac{dx}{dt} \propto x$ i.e. $\frac{dx}{dt} = kx$ where $k > 0$

The solution of DE is given by $\int \frac{dx}{x} = k \int dt$ i.e. $\log x = kt + c$ -------- (1)

The given condition in tabular form are as follows :

Using these conditions in equation (1), we get

$\log x_0 = k\, t_0 + c$ ---------- (2) and

$\log x_1 = k\, t_1 + c$ ---------- (3)

$\therefore$ for elimination of c, subtracting (2) from (1) and (3), we get

Now eliminating k, we get $\frac{\log x - \log x_0}{\log x_1 - \log x_0} = \frac{t - t_0}{t_1 - t_0}$

$$\frac{\log x - \log 5{,}00{,}000}{\log 8{,}00{,}000 - \log 5{,}00{,}000} = \frac{2010-1980}{1990-1980}$$

$$\frac{\log\left(\frac{x}{5{,}00{,}000}\right)}{\log\left(\frac{8}{5}\right)} = \frac{3}{10}$$

i.e. $\log\left(\frac{x}{5{,}00{,}000}\right) = 3\log\left(\frac{8}{5}\right) = \log\left(\frac{8}{5}\right)^3$

$\therefore \left(\frac{x}{5{,}00{,}000}\right) = \left(\frac{8}{5}\right)^3$

i.e. $x = \frac{512}{125}(5{,}00{,}000)$

$x = 20{,}48{,}000$

Conclusion : The population of the town in year 2010 will be 20,48,000.

Experiment No. 18

Problem 2 : A body cools from 140° C to 80° C in 20 min. If the temperature of surrounding is 20° C. when will be temperature of body be 50°C.

Formula : Newton's law of cooling.

Step 1 : Form the differential equation.

Step 2 : Solve the differential equation.

Step 3 : Find the formula for x in terms of t or t in terms of x ork in terms of x and t.

Step 5 : Calculate x for given value of t or t for given value of t or t for given value of t or t for given value of t or t for given value of x or k for given value of x and t.

Tabular form :

Time t:	$t_0=0$	$t_1=20$	t=?
Temperature of body T :	$T_0=140°C$	$T_1=80°C$	T= 50°C
Temperature of medium M	20°C	20°C	20°C
Temperature diff. T.N.	$T_0 - M = 120°C$	$T_1 - M = 60°C$	$T_1 - M = 30°C$

Conclusion : Thus it will take 40 minutes for the body to reach the temperature of 50°C.

Experiment No. 18

Problem 2 : A body cools from 140° C to 80° C in 20 min. If the temperature of surrounding is 20° C. when will be temperature of body be 50°C.

Solution : Let T°C be the temperature of the body at time t and M°C be the temperature of surrounding. Therefore by Newton's law of cooling.

$\therefore \frac{dT}{dx} \propto (T - M)$ i.e. $\frac{dT}{dt} = k(T - M)$, where, $k < 0$

$\therefore \int \frac{dt}{T-M} = k \int dt$

$\therefore$ The solution of the DE is $\log(T - M) = kt \pm c$ ------------ (1)

The given conditions are as follows :

Using these conditions in equation (1), we get

$\log(T_0 - M) = kt_0 + c$ ------------ (2) and

$\log(T_1 - M) = kt_1 + c$ ------------ (3)

For elimination of C_1 subtract (2) from (1) and (3), we get

$\log(T - M) - \log(T_0 - M) = k(t - t_0)$ ------------ (4)

And $\log(T_1 - M) - \log(T_0 - M) = k(t_1 - t_0)$ ---------- (5)

Now to eliminate k, divide equation (4) by (5),

We have

$$= \frac{\log(T-M) - \log(T_0-M)}{\log(T_1-M) - \log(T_0-M)} = \frac{t-t_0}{t_1-t_0}.$$

$$\therefore \frac{\log(30) - \log(120)}{\log 60 - \log 120} = \frac{t-0}{20-0}$$

$$\therefore \frac{t}{20} = \frac{\log 4}{\log 2} = \frac{2\log 2}{\log 2}$$

$\therefore t = 40$

Conclusion : Thus it will take 40 minutes for the body to reach the temperature of 50°C.

Experiment No. 19

Aim : Expected value, variance and S. D. of random variable.

Problem 1 : A box contains 12 items of which 3 are defective. A sample of 3 items is selected from the box. Find the probability that, in the selection

1) At the most one is defective
2) 1 or 2 are defective
3) μ
4) σ^2
5) σ

Formula :

i. $P\,(x = 0) = {}^3C_0 \times {}^9C_3 / {}^{12}C_3$

ii. $\mu = \sum x_i p_i$

iii. $\sum x_i^2 P_i = E(x^2)$

iv. $\delta^2 = \sum x_i^2 P_i - \mu^2$

v. $\delta = \sqrt{v(x)}$

Tabular Form :

X=x;	0	1	2	3
P (x)= P;	84/220	108/220	27/220	1/220

Conclusion : The probability that in the selection

i. at the most one is defective is $\frac{48}{55}$

ii. 1 or 2 defective is $\frac{27}{44}$

iii. $\mu = \frac{3}{4}$

iv. $\sigma^2 = 0.46$

v. $\sigma = 0.678$

Experiment No. 19 RHS

Aim : Expected value, variance and S. D. of random variable.

Problem 1 : A box contains 12 items of which 3 are defective. A sample of 3 items is selected from the box. Find the probability that, in the selection

1) At the most one is defective
2) 1 or 2 are defective
3) μ
4) σ^2
5) σ

Solution : Let the random variable x denotes defective items in the sample space. In all there are 12 items, of which 3 are defective items, of which 9 are non defective.

$\therefore$ x takes the values 0,1,2 or 3.

Now,

i. $P\ (x=0) = {}^3C_0 \times {}^9C_3 / {}^{12}C_3 = \frac{1\times9\times8\times7}{1\times2\times3} \times \frac{1\times2\times3}{12\times11\times10} = \frac{84}{220}$

$P\ (x=1) = {}^3C_1 \times {}^9C_2 / {}^{12}C_3 = \frac{3}{1} \times \frac{9\times8}{1\times2} \times \frac{1\times2\times3}{12\times11\times10} = \frac{108}{220}$

$P\ (x=2) = {}^3C_2 \times {}^9C_1 / {}^{12}C_3 = \frac{3}{1} \times \frac{9\times1\times2\times3}{12\times11\times10} = \frac{27}{220}$

$P\ (x=3) = {}^3C_3 \times {}^9C_0 / {}^{12}C_3 = \frac{1\times1\times2\times3}{12\times11\times10} = \frac{1}{220}$

$\therefore$ The probability distribution of x-axis is

i. $P\ (x = 0\ or\ 1) = P(x=0) + P(x=1) = \frac{84}{220} + \frac{108}{220} = \frac{192}{220} = \frac{48}{55}$

ii. $P\ (x = 1\ or\ 2) = P(x=1) + P(x=2) = \frac{108}{220} + \frac{27}{220} = \frac{135}{220} = \frac{27}{44}$

iii. $\mu = \sum x_i p_i = (0)\left(\frac{84}{220}\right) + (1)\left(\frac{108}{220}\right) + 2\left(\frac{27}{220}\right) + 3\left(\frac{1}{220}\right) = \frac{165}{220} = \frac{3}{4}$

iv. $\sum x_i^2 p_i = (0)^2\left(\frac{84}{220}\right) + (1)^2\left(\frac{108}{220}\right) + 2^2\left(\frac{27}{220}\right) + 3^2\left(\frac{1}{220}\right) = \frac{225}{220} = \frac{45}{44}$

$\therefore \sigma^2 = \sum x_i^2 p_i - \mu^2 = \frac{45}{44} - \frac{9}{16} = \frac{324}{704} = 0.46$

v. $\sigma = \sqrt{0.46} = 0.678$:

Conclusion : The probability that in the selection

i. At the most one is defective is $\frac{48}{55}$

ii. 1 or 2 are defective is $\frac{27}{44}$

iii. $\mu = \frac{3}{4}$

iv. $\sigma^2 = 0.46$

v. $\sigma = 0.678$

Experiment No. 19

LHS

Aim : Expected value, variance and S. D. of random variable.

Problem 1 : A box contains 12 items of which 3 are defective. A sample of 3 items is selected from the box. Find the probability that, in the selection

1) At the most one is defective
2) 1 or 2 are defective
3) μ
4) σ^2
5) σ

Formula :

i. $P(x = 0) = {}^3C_0 \times {}^9C_3 / {}^{12}C_3$

ii. $\mu = \sum x_i p_i$

iii. $\sum x_i^2 P_i = E(x^2)$

iv. $\delta^2 = \sum x_i^2 P_i - \mu^2$

v. $\delta = \sqrt{v(x)}$

Tabular Form :

X=x;	0	1	2	3
P (x)= P;	84/220	108/220	27/220	1/220

Conclusion : The probability that in the selection

i. at the most one is defective is $\frac{48}{55}$

ii. 1 or 2 defective is $\frac{27}{44}$

iii. $\mu = \frac{3}{4}$

iv. $\sigma^2 = 0.46$

v. $\sigma = 0.678$

Experiment No. 19

Problem 2 : In a game, a person is paid Rs. 10 if he gets all heads or all tails when three coins are tossed and he will pay Rs. 5 if he get either one or two heads. What can be expect to win on average per game? Also find the variance and S.D.

Solution : $S = \{HHH,\ HHT,\ HTH,\ HTT,\ THH,\ THT,\ TTH,\ TTT\}$

$\therefore\ n(s) = 8$

The random variable x takes the values 10 or -5.

Now, $P(x = 10) = P\{HHH, TTT\} = \frac{2}{8} = \frac{1}{4}$

$P(x = 10) = P\{HTT, TTH,\ THT,\ HHT,\ THH,\ HTH\}$

$P\ (X = -5) = \frac{6}{8} = \frac{3}{4}$

The probability distribution is :

$\therefore$ Expected value $= E(x) = \sum p_i x_i = -\frac{5}{4} = 1.25$

$v(x) = \sum(x^2) - [E(x)]^2 = \sum p_i x^2 - (\sum p_i x_i)^2 = \frac{175}{4} - \left(\frac{-5}{4}\right)^2$

$v(x) = \frac{175}{4} - \frac{25}{16} = \frac{675}{16} = 42.2$

S.D. $= \sqrt{v(x)} = \sqrt{42.2} = 6.496$

Conclusion : The person will on an average lose Rs. 1.25 per toss of coin variance= Rs. 42.2 and S.D. = Rs. 6.496

<u>Experiment No. 20</u>

LHS

Aim : Binomial Distribution.

Problem 1 : A coin is tossed 5 times. If getting a head is considered a success, find the probability of at least 3 successes.

Formula : The probability of r successes= $P(x = r)$ in n trials = $^{n}C_{r}\, p^{r}\, q^{n-r}$

$r = 0,1,2,\ldots\ldots\ n$

Conclusion : The probability of at least 3 successes is 1/2

Experiment No. 20

Aim : Binomial Distribution.

Problem 1 : A coin is tossed 5 times. If getting a head is considered a success, find the probability of at least 3 successes.

Solution: In a single trial, getting a head is a success $\therefore p = P\,(getting\ a\ head) = \frac{1}{2}$,

Therefore q= 1-p= ½

We have n=5, p=1/2, q= 1/2

By binomial distribution $p(x = r) = {}^5C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{5-r}$

r=0,1,2,3,4,5

p (at least 3 successes)= $p(x \geq 3) = p(x = 3) + p(x = 4) + p(x = 5)$

$\therefore p(3) = {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2 = \frac{10}{32}$

$p(4) = {}^5C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right) = \frac{5}{32}$, $p\,(5) = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$

$\therefore$ p (at least 3 successes)= $\frac{10}{32} + \frac{5}{32} + \frac{1}{32} = \frac{1}{2}$

Conclusion : The probability of at least 3 successes is 1/2

Experiment No. 20

LHS

Problem 2 : A discrete random variable x has mean score equal to 'δ' and variance equal to 2. If the probability distribution is binominal, what is probability when $5 \leq x \leq 6$.

Formula : $p\ (x = r) = {}^{n}C_{r} p^{r} q^{n-r}, r = 0,1,2, \ldots \ldots . n$

$P(5 \leq x \leq 6) = P(x = 5) + P(x = 6)$

$P = 1 - q$

Conclusion : The probability when $5 \leq x \leq 6$ is $\frac{9408}{3^9}$

Experiment No. 20

Problem 2 : A discrete random variable x has mean score equal to 'δ' and variance equal to 2. If the probability distribution is binominal, what is probability when $5 \leq x \leq 6$.

Solution : In Binomial distribution, expected value= np

and p and variance = npq

$\therefore np = 6$ and $npq=2$ $\qquad \therefore 6q = 2$

Hence $q = \frac{2}{6} = \frac{1}{3}$

$\therefore p = 1 - q = \frac{2}{3}$ and $n\left(\frac{2}{3}\right) = 6$ $\qquad \therefore n = 9$

Since ,

$$p\ (x = r) = {}^{n}C_r p^r q^{n-r}, r = 0,1,2, \ldots \ldots . n$$

$$= {}^{9}C_r\, p^r q^{9-r},\ r=0,1,2,3,\ldots.9$$

$$\therefore p(5 \leq x \leq 6) = P\ (x = 5\ or\ 6) = p(x = 5) + p(x = 6)$$

$$= {}^{9}C_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^4 + {}^{9}C_6 \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^3$$

$$= \frac{9\times8\times7\times6}{1\times2\times3\times4} \times \frac{32}{3^9} + \frac{9\times8\times7}{1\times2\times3} = \frac{64}{3^9}$$

$$P(5 \leq x \leq 6) = \frac{1}{3^9}[126 \times 32 + 84 \times 64] = \frac{9408}{3^9}$$

Conclusion : The probability when $5 \leq x \leq 6$ is $\frac{9408}{3^9}$